Firm Dynamics and Financial Development*

Cristina Arellano
Federal Reserve Bank of Minneapolis,
University of Minnesota, and NBER

Yan Bai
Arizona State University

Jing Zhang
University of Michigan

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Abstract

This paper studies the impact of cross-country variation in financial market development on firms' financing choices, growth and default using comprehensive firm-level datasets. We document that in less financially developed economies, small firms grow faster and have lower leverage ratios than large firms. As financial development improves, the differences in growth and leverage of small and large firms shrink, especially for entrant firms. We develop a quantitative model where financial frictions drive firm growth and debt financing through the availability of credit and default risk. We parameterize the model to the firms' financial structure in the data and show that financial restrictions can account for the majority of the difference in growth rates between firms across countries. The model also matches the data in that small and highly leveraged firms are more likely to default in less financially developed economies.

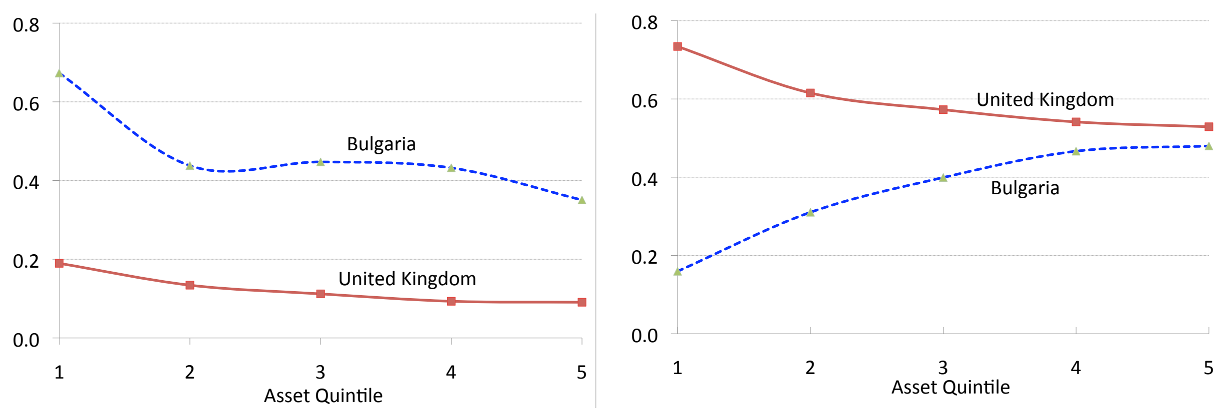
*Arellano: arellano@econ.umn.edu; Bai: yan.bai@asu.edu; Zhang: jzhang@umich.edu. We thank Jacek Rothert for excellent research assistance. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

Financial restrictions can hinder firms' ability to use inputs efficiently and affect firm growth. Recent theoretical models of firm dynamics predict that limited credit makes inefficiently small and young firms grow faster than large firms.[1] However, evidence for the magnitude of these effects in actual firm-level data is scarce.[2] The central goal of this paper is to use cross-country variation in financial market development to evaluate empirically and quantitatively the impact of financial frictions on firms' financing choices and growth rates with firm-level datasets.

Consider two countries with varying financial market development: the United Kingdom and Bulgaria. Figure 1 plots the growth-size and leverage-size relations for firms in these 2 countries.[3] In both countries, small firms grow faster than large firms, but the difference in growth rates is larger in Bulgaria. The difference in leverage ratios across firms and countries is striking. Small firms in Bulgaria have lower leverage ratios than large firms, whereas in the United Kindgom the relation is reversed.

Figure 1: Firm Size, Leverage and Sales Growth



(a) Size and Growth

(b) Size and Leverage

This paper documents that such differences in growth rates and leverage ratios across firms is robust across many countries with varying financial market development. We use comprehensive firm-level data from 27 European countries and focus on the *relative* behavior of firms of different sizes across countries with *varying* financial development, as indicated by the ratio of private credit to GDP and the coverage of credit information for consumers

[1] Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), Quadrini (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007), among others.

[2] One exception is Huynh and Petrunia (2009), who document for Canadian manufacturing firms that financial factors, such as leverage and financial asset, impact growth rates for new firms.

[3] Growth is measured by annual sales growth, leverage is measured by the ratio of total debt to total assets, and size is measured by 5 asset quantiles. For more details on these definitions see Section 2.

and firms. Consistent with theories of financial frictions, we find that small and young firms grow disproportionately faster than large and more mature firms especially in less financially developed countries. We also find that small firms tend to have lower leverage ratios than large firms on average. However as financial markets improve, the leverage ratio of small relative to large firms increases, although by less for the younger firms. The relations among size, growth, leverage and financial development are not only statistically significant but also economically important. For example, we find that a 120 percentage points difference in the ratio of credit to GDP (as found between the United Kingdom and Bulgaria) is associated with a 54 percentage points difference in growth rates between two firms with assets equal to 1% and 0.01% of the economy's assets respectively. Importantly, all these findings are robust to controlling for country, industry, or age-specific characteristics.

Our empirical analysis provides a systematic cross-country investigation of the interactions between financial market development, size, debt financing, and growth for incumbent and entrant firms with extensive datasets that include a large number of small private firms. By controlling for a large set of fixed effects, we identify sharply the implications of financial frictions on the differential growth rate and debt financing across firms. For additional robustness of the results, we also include in the investigation measures that control for the two other leading theories for firm dynamics, which are based on selection mechanisms and mean reversion in the accumulation of factors of production. We find evidence of both theories: countries with strong selection effects have a more negative relation between size and growth and this size-growth relation varies across industries. Even after introducing these controls, financial considerations continue to play a prominent role in the dynamics of firms.

In the second part of the paper, we develop a model to highlight the mechanisms that link firm growth to financial conditions, and perform a counterfactual exercise as well as a quantitative assessment of the theory. Credit restrictions arise in our model because firms can default and lenders incur a fixed cost when issuing debt. A high credit cost induces high default risk, and in turn limits credit, which proxies a low degree of financial market development. Debt is restricted disproportionately for small firms in less financially developed economies and these restrictions make their scale inefficient. These small firms grow faster because they can expand their scale. Hence, in the model small firms in less financially developed economies have disproportionately less debt financing and higher growth rates, as in the data.

The framework is a dynamic stochastic model where firms use a decreasing returns to scale technology to transform capital into output and face uncertain productivity. They finance capital and dividends with debt and profits and have the option to default on their debt. Firms face debt schedules that encode their default risk. These schedules impact firms' debt financing and capital choices. Increasing debt is useful for financing capital and dividends, but larger loans are costly because of higher default risk. Hence, firms prefer to shrink their size and become inefficiently small to avoid excessively large loans, especially after a history of low shocks. Firms can also be small simply because the persistent component of their productivity is low.

The firm-specific debt schedules determine their size, growth, and leverage. Small firms both *unproductive* and *unlucky* are more likely to be inefficient in scale because they face more restricted schedules or are closer to their borrowing limits. Small firms grow faster in response to good shocks because they use the additional output to increase their scale to a more efficient level. Hence, our model matches the first empirical regularity that small firms grow faster than large firms especially in less financially developed economies. Small *unproductive* firms have lower leverage due to their tight debt schedules, but small *unlucky* firms have relatively larger loans due, as they have built up debt after a history of bad shocks. Hence, small firms can have higher or lower leverage than large firms. Nonetheless, as credit costs and default risk increase, the restrictions on loan contracts become so severe for the small unproductive firms that the leverage of small versus large firms decreases. Thus, our model can match the second empirical regularity that the difference in debt financing of small and large firms decreases in less financially developed economies.

We quantitatively evaluate the model implications in rationalizing the cross-sectional financing and growth patterns jointly. We calibrate our model using the firm-level data of Bulgaria and the United Kingdom as representative countries with weak and strong financial market development. Our calibration strategy consists of choosing the parameters capturing the degree of debt enforceability to match the financing patterns observed in the cross section of firms in each country. The calibrated credit cost for Bulgaria equals 0.03% of output for the average firm and restricts credit such that the average leverage ratio equals 0.36. For the United Kingdom this cost is zero, which delivers an average leverage ratio equal to 0.61. We then evaluate the model's predictions on growth rates for firms of different sizes. The model replicates quantitatively the observed patterns of sales growth and firm size in both the Bulgarian and British calibrations.

With our calibrated model, we analyze the consequences of improving the development of financial markets in Bulgaria by reducing the credit cost to the UK level. We find that consistent with the data, following this experiment the size-leverage relation flips from positive to negative, and the size-growth relation becomes flatter. In particular, the difference in growth rates between the smallest and largest firms declines from 11% to 5%, and the difference in leverage ratios increases from −21% to 5%. Better financial market development also increases the output of the small firms by 10%. This exercise also reveals that the cross-country variation in financial development accounts for majority of the differential in growth rates and leverage ratios across firms.

Varying financial markets also has a differential effect on the growth and leverage of entrant versus incumbent firms. The model predicts that in less financially developed economies among entrant firms, the relation of size and growth is more negative than for incumbents and leverage is negatively related to size. For economies with better financial markets, the model predicts that for entrants the relations of growth-size and leverage-size are flat. These predictions are consistent with the empirical evidence: as financial development improves, the difference in sales growth and leverage of small and large entrant firms relative to incumbents shrink.

Finally, the model has sharp implications for default, size, leverage and financial development. In the model, defaulting firms tend to be small and have high leverage ratios before exiting. Moreover, the model predicts that small and highly leveraged firms have a disproportionately higher default probability in less financially developed economies. These additional model implications are confirmed in the panel cross country dataset. We find that the small and highly leveraged firms are disproportionately likely to be in liquidation or bankruptcy in less financially developed countries.

Related Literature

Our empirical findings are novel because we are the first to examine the cross-sectional firm financing and growth patterns simultaneously across countries with a broad coverage of firms. In regard to growth, the cross-section firm-level analyses have considered only one country, as in Rossi-Hansberg and Wright (2007) for the United States.[4] In regard to firms' financing patterns, cross-country comparisons have been studied only for large public firms; Rajan and Zingales (1995) examine G7 countries, and Booth et al. (2001) study 10 developing countries. Public firms, however, constitute a small percentage of firms in all countries, which limits the scope of these previous findings.[5]

The theoretical model is related to the literature that studies the implications of financial frictions on firm growth. Our theory is closest to Cooley and Quadrini (2001), who develop a model where financing restrictions arise from limited commitment in debt contracts. They show that these frictions can potentially deliver large differences in the growth rates between small and large firms. In our paper, we use firm-level data to quantify the extent to which financial considerations impact growth rates. We further concentrate on how differences in

[4]The cross-country analysis of growth has been restricted to industry-level data, as in Rajan and Zingales (1998).

[5]For example, in the United Kingdom less than 2% of firms in our dataset are public firms.

financial market development can explain firm financing and growth patterns across countries. Our paper is also closely related to Albuquerque and Hopenhayn (2004), who analyze the effects of enforcement problems under a full set of state-contingent assets. In our model, we use incomplete markets to allow firms with a history of bad shocks to decrease their value and to allow precautionary savings to play a role.[6]

Apart from financial frictions, the two leading theoretical explanations for why small firms grow faster are based on selection mechanisms and mean reversion in the accumulation of factors of production. Hopenhayn (1992) and Luttmer (2007), for example, propose theories where the growth of small firms reveals a selection effect: small firms tend to exit with bad shocks, and so they grow faster when they survive after good shocks. Rossi-Hansberg and Wright (2007) develop a model where the mean reversion in the accumulation of industry-specific human capital makes small firms grow faster. Nonetheless, theories of firm growth without financial frictions are silent (by construction) regarding the joint financing and growth patterns of firms across countries.

The paper is also related to the literature in corporate finance on the capital structure of firms.[7] Hennessy and Whited (2005) develop a dynamic model of debt financing and show that progressive taxes induce larger firms to use more debt financing. Interestingly, this theory is at odds with the data in the United Kingdom where corporate taxes are progressive, yet the relation between size and leverage is negative. Miao (2005) also studies optimal capital structure of firms in a model with endogenous exit in response to productivity shocks. In his model, firms choose debt only when they enter, yet small firms have higher leverage ratios because their equity value is small. In our model, the firm's debt choice is time varying and the interest rate on debt reflects endogenous default probabilities.

The rest of the paper is organized as follows. Section 2 presents the new empirical findings on firm growth and debt financing across countries with varying financial development. Section 3 introduces and characterizes the model. Section 4 presents the quantitative assessment of the model and counterfactual experiments. Section 5 concludes.

2 Empirical Facts

In this section we first describe the Amadeus database, which provides information on firm-level balance sheets in European countries. We then present our main empirical findings. Small firms grow faster and use less debt financing than large firms. When financial devel-

[6] Quadrini (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007) also study theoretically financial constraints that arise due to informational asymmetries between lenders and entrepreneurs.

[7] See Harris and Raviv (1991) for a comprehensive review.

opment improves, the growth of small relative to large firms decreases, especially for young firms; while the leverage of small relative to large firms increases, but by less for young firms. We finally establish the robustness of our results by controlling for alternative explanations and exploring various years.

2.1 Data Description

The data source is Amadeus, which is a comprehensive European database. Amadeus contains financial information of over 7 million private and public firms in 38 European countries covering all sectors in the economy. We focus on the firms' balance sheet data in 2004 and 2005.[8] We measure firm size by the book value of the total assets of the firm in 2004.[9] To measure debt financing, we compute the firm's leverage ratio in 2004. Leverage is defined as the ratio of total debt and total asset. Total debt includes short-term and long-term debt as well as short-term loans from suppliers.[10] Firm growth is measured by the growth rate of sales from 2004 to 2005.

To clean the dataset, we first exclude firms in the financial and government sectors following Rajan and Zingales (1995). We restrict the sample to firms that report positive assets, non-negative liabilities in 2004 and non-negative sales in both 2004 and 2005. We next remove firms with growth rates or leverage ratios in the top one percentile in each country. Finally, we drop the countries that have less than 1000 observations after cleaning.[11] These criteria leave us with about 2.6 million firms in 27 countries: Belgium, Bosnia and Herzegovina, Bulgaria, Croatia, Czech Republic, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, the Netherlands, Poland, Portugal, Romania, Russian Federation, Serbia, Slovakia, Spain, Sweden, Ukraine, and the United Kingdom.[12] In the appendix we show that the datasets for these 27 countries are quite representative of the universe as reported by the European Commission. The fraction of small firms does vary some across countries, but this variation is uncorrelated with our measures of financial development.

[8] We focus on 2004 and 2005 as a benchmark because these two years offer the most extensive coverage and are less affected by backlog reporting.

[9] We favor using book value as opposed to market value of assets because less than 1% of the firms in the sample are public firms.

[10] In a previous draft of this paper, we use a broader definition of debt which included all liabilities. The relation between size-leverage and financial development is robust to this alternative definition. However, the unconditional correlation between size and leverage was negative.

[11] The appendix contains more details about the data cleaning procedure.

[12] The countries that are excluded in this analysis are Austria, Belarus, Cyprus, Denmark, Liechtenstein, Luxembourg, Macedonia, Moldova, Monaco, Norway, and Switzerland. The threshold of 1000 is not critical: only the Switzerland will be included in our sample if 500 is used as the threshold instead.

The development of financial markets in these 27 countries is measured using two statistics. The first one is the average private credit to GDP ratio over 2000–2004 taken from the *World Development Indicators*. Higher ratios of private credit to GDP indicate better financial development. The second measure is the coverage of credit bureaus. Credit bureaus in countries track the loans and defaults of individuals and firms and facilitate lending by banks and financial institutions. The statistic we use is the percentage of adults that are included in the public and private credit bureaus in 2004 in each country.[13] Larger credit bureau coverage indicates better financial development because it implies that it is easier for financial intermediaries to make loans when credit information of borrowers is available. Credit bureau coverage is taken from the Doing Business publications of the World Bank.

Table 1 reports descriptive statistics for the firm-level datasets and the two measures of financial markets development for each country. Countries are ordered by their level of private credit to GDP. The table shows that the variability of financial development is large across these 27 countries. For example, the private credit to GDP ratio is 143% in the Netherlands and only 18% in Russia; the credit bureau coverage is 98% in Sweden and 0% in Croatia. As expected, these two financial development indices are highly correlated in our sample with a correlation of 0.81.

The mean and median level of assets for firms in each country are reported for 2004 in terms of thousand current euros in the table. Firm asset levels vary across countries, and they tend to be larger in countries with more developed financial markets. Moreover, the distribution of firms in all countries is highly skewed, as the mean asset levels are much larger than the median asset levels.[14] We also report the average leverage ratio and the average net growth rate (CPI-adjusted) across all firms in each country. Both mean leverage and mean growth vary substantially across countries. The mean leverage ratio is 0.6 in the United Kingdom, but only 0.01 in Hungary; the mean net growth rate is 7% in the Netherlands, but 66% in Estonia. The table also reports the number of firms in the clean datasets of each country, which is the sample used in the main regression results that follow.

Overall, these aggregate statistics are systematically related to financial market development. First, firms in countries with more developed financial markets tend to have larger leverage ratios. The cross-country correlation of mean leverage and private credit to GDP is 0.19, and the correlation of mean leverage and credit bureau coverage is 0.20. Second, firm growth is on average smaller in countries with better financial development. The cross-

[13]We use the 2005 statistics for Iceland and the United Kingdom because this statistic is not available for them in 2004.

[14]Cabral and Mata (2003) find the similar pattern of firm size distribution in the universe of Portuguese manufacturing firms. Quintin (2008) emphasizes that enforcement constraints might account for the difference in firm size distribution across countries.

Table 1: Summary of Firm-Level Datasets and Financial Development

	Firm-Level Datasets					Financial Development %	
	Mean Asset	Median Asset	Mean Leverage	Mean Growth	No. Firms	Credit Coverage	Credit to GDP
Netherlands	263724	24124	0.24	0.07	5077	65	143
United Kingdom	71260	849	0.60	0.12	67748	76	142
Portugal	17939	787	0.51	0.07	19784	64	138
Iceland	2017	85	0.59	0.55	4096	100	120
Germany	198267	3205	0.44	0.09	20225	86	116
Ireland	128762	3417	0.39	0.18	1807	100	115
Spain	5694	365	0.22	0.18	437405	39	109
Sweden	12296	323	0.36	0.13	93116	98	89
France	7621	220	0.32	0.07	637764	2	87
Italy	7740	659	0.14	0.10	414447	57	81
Belgium	22789	393	0.46	0.04	41995	53	75
Greece	9484	1535	0.50	0.10	20191	11	60
Finland	16201	284	0.40	0.13	26154	15	60
Estonia	560	37	0.33	0.66	34187	10	46
Croatia	656	115	0.46	0.01	6922	0	42
Slovakia	9649	1556	0.39	0.15	4511	18	38
Hungary	375	30	0.01	0.54	207207	3	38
Czech Republic	3436	221	0.27	0.37	40429	25	37
Latvia	3712	588	0.59	0.30	3142	1	34
Bosnia	2791	379	0.47	0.19	2660	16	33
Poland	23451	3624	0.38	0.03	8044	38	27
Serbia	1300	70	0.52	0.08	29385	0	27
Bulgaria	1463	91	0.36	0.47	17894	1	22
Lithuania	8556	1738	0.49	0.40	2237	12	19
Russia	2484	55	0.43	0.64	237639	0	19
Ukraine	6618	705	0.28	0.04	15594	0	18
Romania	326	14	0.00	0.74	269044	0	11

country correlation of mean growth and private credit to GDP is -0.37, and the correlation of mean growth and credit bureau coverage is -0.26. Third, firms in countries with more developed financial markets are larger. The correlation of the mean asset level and private credit to GDP equals 0.6, and the correlation of the mean asset level and credit coverage is 0.53.

Now consider the relation of firm size with debt financing and growth across countries. As motivated in the introduction, the size-leverage relation is more negative, while the size-growth relation is less negative, in the United Kingdom than in Bulgaria. We examine this unconditional relation of leverage and sales growth with firm size for all the sample countries. In particular, we focus on how these relations vary with financial development across countries. To this end, we divide firms in each country into 5 quintiles according to their 2004 assets and compute the average leverage in 2004 and the average real sales growth from 2004 to 2005 for each asset quintile. Table 2 reports these statistics.

We find that across these 27 European countries, small firms have, on average, higher growth rates than large firms. In terms of leverage, the pattern is mixed; small firms have higher leverage than large firms in about a half of our sample countries. The unconditional correlations of financial development and the difference in growth and leverage of firms in the largest quintile and in the smallest quintile confirm the Bulgaria-UK patterns. The correlations of the growth difference with private credit to GDP and the credit coverage equal 0.45 and 0.37, respectively. The correlations of the leverage difference with private credit to GDP and with credit coverage equal -0.16 and -0.24, respectively. That is, countries with better financial development have a less negative size-growth relation but a more negative (or less positive) size-leverage relation. In the next subsection, we further examine these relations using a detailed set of controls.

Table 2: Unconditional Leverage and Growth across Asset Quintiles

	Leverage					Growth				
Asset quintile	1	2	3	4	5	1	2	3	4	5
Belgium	0.45	0.47	0.48	0.46	0.45	0.08	0.07	0.07	0.06	0.07
Bosnia	0.52	0.52	0.52	0.45	0.33	0.21	0.18	0.21	0.24	0.16
Bulgaria	0.16	0.31	0.40	0.47	0.48	0.76	0.51	0.52	0.50	0.42
Croatia	0.39	0.45	0.49	0.49	0.47	0.06	0.03	0.03	0.01	0.07
Czech Republic	0.16	0.23	0.27	0.34	0.33	0.83	0.43	0.31	0.21	0.20
Estonia	0.17	0.28	0.37	0.42	0.43	1.27	0.72	0.57	0.50	0.58
Finland	0.39	0.41	0.42	0.41	0.35	0.19	0.13	0.11	0.13	0.13
France	0.31	0.33	0.33	0.33	0.32	0.16	0.09	0.07	0.06	0.07
Germany	0.46	0.51	0.48	0.39	0.34	0.19	0.09	0.10	0.08	0.08
Greece	0.47	0.49	0.50	0.51	0.52	0.26	0.14	0.11	0.10	0.09
Hungary	0.00	0.00	0.01	0.01	0.05	1.08	0.84	0.44	0.31	0.30
Iceland	0.61	0.58	0.57	0.59	0.60	0.98	0.58	0.38	0.48	0.61
Ireland	0.46	0.36	0.37	0.36	0.39	0.23	0.24	0.18	0.20	0.17
Italy	0.08	0.09	0.10	0.12	0.30	0.16	0.13	0.11	0.10	0.10
Latvia	0.61	0.62	0.60	0.58	0.54	0.58	0.37	0.31	0.29	0.37
Lithuania	0.51	0.51	0.48	0.50	0.44	0.99	0.35	0.27	0.31	0.25
Netherlands	0.14	0.19	0.30	0.29	0.25	0.10	0.08	0.08	0.08	0.09
Poland	0.43	0.44	0.39	0.37	0.28	0.04	0.04	0.05	0.07	0.08
Portugal	0.53	0.50	0.50	0.52	0.48	0.16	0.09	0.09	0.08	0.08
Romania*	0.00	0.00	0.00	0.00	0.00	1.85	1.07	0.74	0.48	0.33
Russia	0.34	0.44	0.47	0.47	0.44	1.48	0.89	0.73	0.62	0.54
Serbia	0.43	0.53	0.56	0.57	0.49	0.34	0.35	0.25	0.18	0.17
Slovakia	0.40	0.43	0.39	0.37	0.34	0.27	0.17	0.15	0.16	0.15
Spain	0.16	0.21	0.23	0.21	0.30	0.34	0.19	0.17	0.19	0.23
Sweden	0.29	0.35	0.39	0.40	0.36	0.17	0.13	0.12	0.12	0.15
Ukraine	0.32	0.25	0.24	0.27	0.31	0.20	0.10	0.16	0.20	0.22
United Kingdom	0.73	0.62	0.57	0.54	0.53	0.22	0.17	0.14	0.12	0.12

*: In all Romanian firm balance sheets, total liability comes solely from the entry of "other liability", and the entry of debt is zero.

2.2 Cross-Country Empirical Findings

Our hypothesis is that in countries with more developed financial markets, small firms have higher leverage ratios and lower growth rates relative to large firms. Therefore, we pool all the countries together and estimate two regressions of the following form:

$$Leverage_{k,c}(\text{or } Growth_{k,c}) = \beta_0 + \beta_1 Size_{k,c} + \gamma_1 Size_{k,c} \times FD_c + Dummy + \nu_{k,c}, \tag{1}$$

where c denotes the country, and k the firm. The dependent variable is the firm's leverage ratio for the leverage regressions and the firm's sales growth rate for the growth regressions. $Size_{k,c}$ is the log of the share of firm k's assets in the total assets of country c. Given the highly skewed firm size distribution, we use the log of the firm asset share as firm size. FD_c corresponds to the two measures of financial development in country c, namely, private credit over GDP and coverage of credit registries. The term *Dummy* corresponds to fixed effects at the country×industry×age level.

The regression specification controls for country-specific effects, 2-digit industry-specific effects, and 7 age-group-specific effects. Country effects control for any country characteristic, for instance, business cycles, institutional quality, the legal system, the political system, and many others. Industry effects are at the 2-digit level constructed with NACE codes. They control for any inherent features of industries, including capital intensity, competition structure, liquidity needs, and tradability. The 7 age groups are constructed at 5-year intervals up to 30 years and a final group for firms with age greater than 30 years. Age effects control for any inherent life cycle features of firms, such as market share and technological development.

As discussed in Rajan and Zingales (1998), the use of fixed effects enables us to control for a much wider array of omitted variables. These dummy variables capture the peculiar features of each age group within each sector of each country, such as the particular technological characteristics or specific tax treatments varying at the country×industry×age level. Only additional explanatory variables that vary within each of the industry-country-age groups need be included. These are firm size and the primary variable of interest, the interaction between firm size and financial market development. According to our hypothesis, we must find the coefficient estimate for the interaction between size and financial development to be negative in the leverage regression and to be positive in the growth regression.

Table 3 reports the regression results using the two measures of financial development. The first two columns report the leverage regressions, and the last two columns report the growth regressions. We report the coefficient on firm size and the coefficient on the interaction term between firm size and financial market development in the table. The standard errors of the regression coefficients are reported in parentheses and are robust to heteroskedasticity throughout the paper.

Table 3: Firm Leverage, Growth and Financial Development

	Leverage		Sales Growth	
	Private Credit to GDP	Credit Bureau Coverage	Private Credit to GDP	Credit Bureau Coverage
Size	0.021***	0.018***	-0.134***	-0.082***
	(0.0002)	(0.0001)	(0.0016)	(0.0010)
FD×*Size*	-0.006***	-0.005***	0.097***	0.051***
	(0.0002)	(0.0002)	(0.0013)	(0.0008)
Adjusted R^2	0.28	0.28	0.06	0.06
Number of observations	2621201	2621201	2621201	2621201

Notes: All regressions have a fixed effect at the country×industry×age level. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1%.

Let's start with the regression that analyzes the size-leverage relation. The estimated coefficient on the interaction variable is negative as expected and statistically significant at the 1% level under both measures of financial market development. The coefficient estimate on size is positive and statistically significant under both measures. Thus, smaller firms have on average lower leverage ratios than large firms, other things being equal. Moreover, when private credit to GDP or credit bureau coverage increases, the leverage ratios of small firms relative to large firms increase.

The interaction term is similar to a second derivative. To interpret its magnitude, let's look at the regression with private credit to GDP and compare a small firm with an asset share equal to 0.01% to a large firm with an asset share equal to 1% in Bulgaria and the United Kingdom. The leverage difference between these comparable small and large firms is 3.3 percentage points higher in the UK than in Bulgaria, as private credit to GDP is higher in the UK by about 120 percentage points. These numbers are economically significant given that the mean leverage ratio for Bulgaria equals 0.36.

Let's next look at the regressions that analyze the size-growth relation. Size continues to be a significant determinant; smaller firms grow faster overall. The estimated coefficients on the interaction term are positive as expected and statistically significant at the 1% level for both measures of financial development. That is, the growth difference between small and large firms decreases with both private credit to GDP and credit bureau coverage. We can interpret the coefficient on the interaction of private credit to GDP and size as follows. The difference in growth rates of a small firm with an asset share equal to 0.01% relative to a large firm with an asset share equal to 1% is 54 percentage points less in the United Kingdom than in Bulgaria.

We now examine whether the relations of size, growth and leverage are different for newly established firms relative to older firms across countries. Cooley and Quadrini (2001) document that younger firms tend to grow faster than older firms even conditional on size. Here we want to test whether a country's financial development influences differently the dynamics and financing patterns of young firms. We pool all the countries together and estimate the following regressions:

$$Leverage_{k,c} \text{ (or } Growth_{k,c}) = \beta_0 + \beta_1 Size_{k,c} + \gamma_1 Size_{k,c} \times FD_c \tag{2}$$

$$+\beta_2 Entry_{k,c} \times Size_{k,c} + \gamma_2 Entry_{k,c} \times Size_{k,c} \times FD_c + Dummy + \nu_{k,c},$$

where $Entry_{k,c}$ is a dummy variable that equals one for new entrants, defined as firms with age less than or equal to two years. In a country with financial development FD_c, the difference in the slopes of the size-leverage (size-growth) relation for entrants relative to incumbent firms is governed by $\beta_2 + \gamma_2 FD_c$. Across countries, the response of this difference to financial development is governed by γ_2; a positive γ_2 implies that this difference in the slopes rises with the degree of financial development.

Table 4: Entrant Firm Leverage, Growth and Financial Development

	Leverage		Sales Growth	
	Private Credit to GDP	Credit Bureau Coverage	Private Credit to GDP	Credit Bureau Coverage
Size	0.022***	0.018***	-0.113***	-0.071***
	(0.0002)	(0.0001)	(0.0016)	(0.0010)
FD×Size	-0.007***	-0.005***	0.083***	0.049***
	(0.0002)	(0.0002)	(0.0012)	(0.0008)
Entry×Size	-0.002***	-0.001***	-0.086***	-0.039***
	(0.0001)	(0.0000)	(0.0011)	(0.0004)
Entry×FD×Size	0.001***	0.001***	0.072***	0.034***
	(0.0001)	(0.0002)	(0.0012)	(0.0008)
Adjusted R^2	0.28	0.28	0.07	0.06
Number of observations	2621201	2621201	2621201	2621201

Notes: All regressions have a fixed effect at the country×industry×age level. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1%.

Table 4 reports the estimated coefficients. The coefficients on size and the interaction between size and financial development are similar to those in the main leverage and sales growth regressions. Thus, the cross-country size-leverage and size-growth relations of incumbent firms are similar to those of all firms. In the sales growth regressions, β_1 and β_2 are negative, implying that small firms grow faster than large firms, especially for entrant firms.

Moreover, γ_1 and γ_2 are positive. This implies that as financial development improves, the growth difference between small and large firms decreases, especially for entrant firms.

In the leverage regressions, β_1 is positive and β_2 are negative. This implies that in less financially developed countries, small firms have lower leverage ratios than large firms, but the difference in leverage between large and small firms is smaller for entrants than for incumbents. Moreover, γ_1 is negative and γ_2 is positive. This means that as financial development improves, small firms increase their leverage relative to large firms but by less for entrant firms than incumbent firms.

2.3 Robustness Tests

In this section we provide robustness on the main results. We first examine whether our findings are robust to controlling for alternative explanations for the negative relation between firm size and growth. To do so, we add in the main regressions three additional interaction terms of size with the country's mean sales growth, GDP per capita, and two-digit industry categories. We then examine our regression results for alternative time periods, namely the years 1999–2003. Extensive analysis in both dimensions confirms the robustness of our results.

2.3.1 Robustness to Other Explanations

One important theoretical explanation of the growth-size relation is the *selection* theory: small firms are more likely to exit under adverse shocks and thus tend to have higher growth rates conditional on survival. If selection differs across countries, one concern is whether our results are robust when we control for such variation. We proxy the degree of selection by the mean growth rate of firms in each country because this theory implies that average firm growth should be higher in countries where selection is more important. Specifically, we add an interaction term between firm size and mean growth to the main regressions.

In a recent work, Rossi-Hansberg and Wright (2007) propose another theory for the relation between firm size and growth based on mean reversion in the accumulation of factors. In their model the growth difference between small and large firms is larger in sectors that use physical capital more intensively. They also document that in the United States, the growth rate of firms declines faster with size in the manufacturing sector than in the service sector. To control industry, we add an additional interaction term between firm size and two-digit industry categories to the main regressions in addition to the interaction between size and mean growth.

Finally, we add the interaction term between firm size and the country's GDP per capita.

This variable allows for the relation of size with sales growth to vary with the log of the country's GDP per capita. In table 5 we report the results with all three interaction terms added in the regressions.[15] In the table we report results when financial development is measured by private credit-GDP. Results with credit bureau coverage are similar and can be found in the appendix.

Table 5: Robustness with Additional Interactions

	Leverage		Sales Growth	
FD×Size	-0.011***	-0.011***	0.014***	0.008***
	(0.0005)	(0.0005)	(0.0013)	(0.0013)
Entry×Size		-0.001***		-0.085***
		(0.0001)		(0.0011)
Entry×FD×Size		0.001***		0.072***
		(0.0001)		(0.0012)
Mean Growth×Size	0.003***	0.003***	-0.134***	-0.121***
	(0.0006)	(0.0006)	(0.0024)	(0.0024)
GDP per capita×Size	0.004***	0.004***	0.002***	0.002***
	(0.0002)	(0.0002)	(0.0007)	(0.0007)
Industry×Size	Yes	Yes	Yes	Yes
Adjusted R^2	0.28	0.28	0.06	0.07
Number of observations	2621201	2621201	2621201	2621201

Notes: All regressions have a fixed effect at the country×industry×age level. The standard errors reported in parentheses are robust to heteroskedasticity. *FD* is the private credit to GDP ratio. *** denotes significant at 1%.

In the growth regression, we find that the coefficient of the interaction between firm size and mean growth is significantly negative as expected by the selection theory. The size and growth relation varies across industries: the coefficients of the interaction between firm size and industry dummies are significantly positive for some industries and significantly negative for others. As the income level rises, the negative growth and size relation tends to be less pronounced. Nevertheless, even after we control for selection, industry variation and GDP per capita, the coefficients of the interaction between firm size and financial market development continue to be significant and positive as in the main regressions. That is, small firms tend to grow relatively faster than large firms in less financially developed countries. Moreover, the coefficients of the interaction terms with the entry dummy are almost the same as those in Table 4.

We also conduct similar robustness tests on the leverage regressions. The results are also reported in Table 5. The estimated coefficients on the interaction terms of firm size and private credit to GDP have the same sign and the same significance under all of these

[15]The sign and significance of these three additional interactions are similar if we add them to the main regression one by one.

alternative specifications as in the main regressions. Moreover, the magnitude of the coefficient on this key interaction term doubles when we add these three additional interaction variables. The same is true for the estimated coefficients on the interaction of firm size and credit bureau coverage. Again, the coefficients of the interaction terms with the entry are fairly consistent with those in Table 4.

We conduct additional sensitivity analysis by adding in regression (1) the interaction terms between firm size and the following variables at the country level: a measure of investor protection, stock market capitalization, and macroeconomic volatility. The results reported in Table 3 are robust when we include these additional controls too. For details, see the appendix.

2.3.2 Robustness to Other Years

The benchmark regression results use data from 2004 and 2005 because Amadeus offers the most extensive and stable coverage of firm-level balance sheet information for many countries in these two years. Though the data coverage for some earlier years is limited, in this section we explore these data and show that our findings are robust across time. Basically, we perform the same regression analysis of leverage and sales growth for 1999, 2000, 2001, 2002 and 2003. We find that the coefficients on size and the interaction between size and financial development are similar across specification (1) and (2). We thus report the regression coefficients of specification (2) in Table 6 with the private credit to GDP ratio as the measure of financial development.

The size-leverage regression results are reported in the upper panel. Across all years, the coefficients of size and the interaction between size and financial development are similar to those in our benchmark result. Thus, our finding about the size-leverage relation across countries with different financial development is robust over time. The coefficients of the interaction terms with the entry dummy are also similar to those in the benchmark year, except for 2002.[16] The size-growth regressions, reported in the lower panel, feature similar coefficients on all variables to those in the benchmark year for all the sample years. This implies that our finding for the relations of firm size, growth, and financial development are also robust over time.

The results using credit bureau coverage as a measure of financial development are reported in the appendix. We find that across all the sample years, the size-growth regression generates all the coefficients similar to our main results. A similar finding holds for the size-leverage regression, except for the coefficient of the triple interaction term γ_2 in three of

[16]In 2002, entrant firms in Ukraine have a very different leverage pattern than for other years. This country drives the different coefficient for entrants.

the years.

Table 6: Robustness with Additional Years

	1999	2000	2001	2002	2003
			Leverage		
Size	0.032***	0.033***	0.034***	0.033***	0.020***
	(0.0002)	(0.0002)	(0.0003)	(0.0002)	(0.0002)
FD×*Size*	-0.016***	-0.017***	-0.019***	-0.018***	-0.007***
	(0.0003)	(0.0003)	(0.0004)	(0.0003)	(0.0002)
Entry×*Size*	-0.001***	-0.002***	-0.002***	0.001***	-0.002***
	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0001)
Entry×*FD*×*Size*	0.001***	0.001***	0.001***	-0.001***	0.002***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0002)
Adjusted R^2	0.20	0.17	0.17	0.21	0.22
			Sales Growth		
Size	-0.151***	-0.153***	-0.156***	-0.145***	-0.143***
	(0.0030)	(0.0026)	(0.0026)	(0.0027)	(0.0020)
FD×*Size*	0.128***	0.123***	0.124***	0.112***	0.104***
	(0.0025)	(0.0021)	(0.0022)	(0.0022)	(0.0015)
Entry×*Size*	-0.074***	-0.078***	-0.085***	-0.088***	-0.100***
	(0.0021)	(0.0021)	(0.0021)	(0.0018)	(0.0017)
Entry×*FD*×*Size*	0.060***	0.065***	0.071***	0.072***	0.084***
	(0.0022)	(0.0021)	(0.0022)	(0.0019)	(0.0017)
Adjusted R^2	0.08	0.08	0.08	0.07	0.08
Number of observations	1282817	1409644	1595747	1857510	2124784

Notes: All regressions have a fixed effect at the country×industry×age level. The standard errors reported in parentheses are robust to heteroskedasticity. *FD* is the private credit to GDP ratio. *** denotes significant at 1%.

In summary, we find that small firms use disproportionately less debt financing and grow disproportionately faster than large firms in countries with worse credit bureau coverage and lower ratios of private credit to GDP. As financial development rises, the differences in the size-leverage and size-growth slopes of entrant and incumbents both rise. These empirical findings are robust and provide a comprehensive picture of the relations of financial market development with financing and growth across firms and across countries.

3 Model Economy

To study theoretically firms' financing choices and dynamics, this section presents a dynamic model of heterogeneous firms that face default risk. The model builds on Cooley and Quadrini (2001) and Albuquerque and Hopenhayn (2004) while incorporating differential degrees of financial market development across economies. In the model, entrepreneurs decide on the level of capital and debt financing for their firms. Credit restrictions arise because debt is unenforceable and firms can default. Lenders offer firm-specific debt schedules that compensate for default risk and for a fixed credit cost they incur when issuing debt. We proxy the degree financial market development with the size of the credit cost; large costs induce high default risk and thus limit the economy-wide credit.

3.1 Firms

Entrepreneurs in the economy are infinitely lived and have access to a mass one of risky project opportunities, which we refer to as firms. Each entrepreneur owns at most one firm and decides on entry, exit, production, and financing plans to maximize the present value of dividends.

An operating firm starts the period with capital K and debt B_R. It produces output with a stochastic decreasing returns technology with capital as input: $y = zK^{\alpha}$, where $0 < \alpha < 1$ and the productivity of the project z follows a Markov process given by $f(z', z)$. It finances the new capital K' and dividends D with internal funds which consist of the firm's output net of debt repayment $zK^{\alpha} - B_R$ and with external funds by acquiring a new loan B'. We define the leverage of this firm as the ratio of total debt due to capital installed B_R/K if $B_R \geq 0$. If the firm starts with assets $B_R < 0$, the firm has no liabilities due, and thus its leverage ratio is equal to zero.

Debt contracts are not enforceable as entrepreneurs can default on their debt and exit. After default, the entrepreneurs have probability θ to restart a new project with the same productivity z. Debt contracts (B', B'_R) are firm specific and incorporate the firm specific default risk. Let $\Omega(K', z)$ denote the set of debt schedules available to a firm with next-period capital K' and productivity z. Each contract $(B', B'_R) \in \Omega(K', z)$ maps a current loan B' to a repayment amount B'_R.

The timing of decision for an operating firm within the period is as follows. At the beginning of the period, δ fraction of firms exit exogenously. All surviving firms receive their shocks. An entrepreneur with capital K, debt B_R, and shock z decides whether or not to default. If the entrepreneur repays his debt, he chooses a new loan, capital for the following period, and dividends. If the entrepreneur defaults, the firm exits. Whenever a firm exits, a new firm enters with zero assets and the same productivity as the exiting firm.

We lay out the recursive formulation for the entrepreneur operating a firm. Upon observing the shock realization, the entrepreneur decides whether to default by comparing the default value V^d with the repayment value V^c:

$$V(K, B_R, z) = \max_{d \in \{0,1\}} (1 - d)V^c(K, B_R, z) + dV^d(z), \tag{3}$$

where $V(K, B_R, z)$ denotes the present value of the firm to the entrepreneur. The default decision can be represented by a binary variable $d(K, B_R, z)$ that equals 1 if default is chosen and 0 if repayment is chosen. The default value is given by

$$V^d(z) = \theta V^e(z),$$

where $V^e(z)$ denotes the value of a potential entrant with productivity z.

If he repays his debt, the entrepreneur keeps his project in operation and decides on production and financing. The entrepreneur chooses capital K', dividends D, and a loan contract (B', B'_R) from the available schedule $\Omega(K', z)$ to maximize the repayment value:

$$V^c(K, B_R, z) = \max_{D, K', (B', B'_R) \in \Omega} D + \beta(1 - \delta) EV(K', B'_R, z') \tag{4}$$

subject to a non-negative dividend condition given by

$$D = zK^\alpha - B_R + B' - K' \geq 0, \tag{5}$$

where $\beta < 1$ denotes the discount rate of the entrepreneur and expectation is taken over the conditional probability distribution of z'.[17] $V^c(K, B_R, z)$ is increasing in K and decreasing in B_R, and $V^d(z)$ is independent of these variables. Thus, default is more attractive for firms with smaller capital and larger debt due.

Optimal debt is determined by trading off costs and benefits of various loans within the set of contracts offered. Debt is beneficial for financing investment. Debt can also be used for dividends, which is attractive when loans are cheap and entrepreneurs discount the future heavily. In addition, debt can be used to relax the non-negative dividend condition when the firm's output is low and the loan due is large. On the other hand, large debt is costly because it can lead firms to default. In particular, a large loan today implies a large repayment the next period that will be costly especially when the productivity shock is low. In this case, income might be so low that the entrepreneur fails to satisfy the non-negative dividend condition, defaults, and loses the project. In anticipation of these possible adverse outcomes, the entrepreneur might have precautionary motives to reduce debt.

In our model with limited enforceability of debt contracts, financing decisions interact with firms' investment. In contrast, in an environment where non-contingent contracts are

[17] In the layout of the model, the probability distribution $f(z', z)$ is assumed to be actual probability distribution which by construction abstracts from risk premia. However, we can also interpret the probability distribution as a risk neutral measure in which case the valuation of firms contains a risk premium component. Such interpretation is common in models of investment under uncertainty as in Pindyck (1991) and also in defaultable bonds pricing models as in Duffie and Singleton (1999).

perfectly enforceable and the non-negative dividend condition is relaxed, firms choose capital such that the expected marginal product of capital equals the risk-free rate:

$$E(z)\alpha K_{fb}(z)^{\alpha-1} = (1+r). \tag{6}$$

We refer to this level of capital $K_{fb}(z)$ as first-best capital for a firm with expected productivity equal to $E(z)$.

Enforcement frictions limit firms' ability to equate the marginal product of capital to the risk free rate. In this environment, investment depends on the set of loan contracts available and is distorted downward. For example, if a firm starts with large debt, it might want to borrow a big loan B' to satisfy the non-negative dividend condition and to keep the investment level at the unconstrained optimal. Nonetheless, given that the set of loans is bounded due to possible defaults, such a big loan might not be offered to the entrepreneur. Hence, the entrepreneur might have to reduce the level of investment, making the project inefficiently small.

The problem for a potential entrant is simple in this model. Whenever an idle entrepreneur receives a project opportunity of productivity z, he decides to undertake the project and enter if the expected value of the project is positive. Thus, the value for a potential entrant is given by

$$V^e(z) = \max\{0, V^c(0,0,z)\}.$$

Note that the new entrant starts with no assets and thus the value conditional on entering is exactly equal to the value of the contract $V^c(0,0,z)$ when K and B_R are equal to zero.

3.2 Debt Schedules

Firms borrow from a debt schedule that depends on their default decisions.[18] Creditors have to pay a fixed credit cost ξ for every loan they offer. Debt schedules $\Omega(K', z)$ include all contracts (B', B'_R) that allow creditors to break even in expected value:

$$B' + \xi = \frac{B'_R(1-\delta)}{1+r}\left(1 - \int d(K', B'_R, z') f(z', z) dz'\right). \tag{7}$$

The left-hand side of equation (7) are the resources creditors spend today. The right-hand side is the expected repayment discounted by the risk-free rate $1+r$.

[18]We generalize the endogenous debt schedules developed in Chatterjee et al. (2007) and Arellano (2008) in their study of unsecure consumer credit and sovereign default by adding an interaction of capital and default risk in the study of firm dynamics.

The availability and the terms of debt contracts are determined by both default risk and the credit cost. Without the credit cost, default risk generates debt restrictions which are proportional to firm productivity. If a firm that invests K' and has productivity z wants a larger transfer B' today, it will need to promise an increasingly larger repayment tomorrow B'_R because of higher default risk. Moreover, for every K' and z the schedule contains an upper bound on borrowing which is associated with excessively high default risk. Firms, however, can improve the availability and terms of their loans by choosing a higher investment K'. Increasing capital makes firms less likely to default and hence allows for larger and more favorable loans.[19]

The credit cost ξ affects the availability of debt through its impact on default risk both at the aggregate and firm level. An economy with large a credit cost features high costs of financing and hence high aggregate default risk. Higher default risk in turn limits the economy-wide availability of credit. Nevertheless, the impact of credit costs ξ within an economy also affects differently firms with varying capital and productivity. In general, high ξ increases default risk disproportionately for firms with low capital and productivity.

One can rationalize the expense of ξ as costs lenders pay to obtain information about the entrepreneur's total debt. Knowing this information is necessary for the lender to correctly assess the probability of default of each entrepreneur. We interpret ξ as the economy's ease to acquire credit information, and it controls financial market development of the model economy. The parameter ξ can be naturally linked to the coverage of credit registries across countries as well as the aggregate level of credit. When ξ is low, credit registries in the economy have wide coverage, and it is very easy and cheap to access credit information. When ξ is large, the lender has to spend some resources to screen the entrepreneur and obtain his debt information.[20] As documented in the empirical section, the coverage of credit registries across countries varies widely, and this variable is linked to the ways firms grow and finance their assets. Thus, our model focuses on variation in ξ to capture differences in financial market development across economies.

Entrepreneurs in our model can also save $B' < 0$. We assume that when the entrepreneur saves creditors do not need to pay ξ as default probabilities are zero. Savings contracts satisfy the following condition regardless of firm characteristic (z, K'):

$$B' = \frac{1-\delta}{1+r} B'_R, \text{ for } B' \leq 0. \tag{8}$$

[19] Our model shares this additional benefit of capital of relaxing borrowing constraints with many models of collateral constraints such as Kyotaki and Moore (1997) and Cagetti and De Nardi (2006).

[20] This specification of credit issuance costs is similar to the one used in Livshits, MacGee, and Tertilt (2008). They document that improvements in credit scoring in the United States are important for understanding the rise in bankruptcies and volume of debt.

3.3 Equilibrium

Before defining the equilibrium of this economy, we make an assumption on the relation between the risk-free rate and the discount factor of entrepreneurs. The assumption imposes that the rate at which entrepreneurs discount the future is higher than the risk-free rate. This condition can be interpreted as a general equilibrium property of economies with lack of enforcement and incomplete markets.[21]

Assumption 1 The risk-free rate r is such that $1/\beta - 1 > r > 0$.

The model delivers an endogenous distribution of firms, denoted by $\Upsilon(K, B_R, z)$, which depends on the decisions of firms to borrow and invest. Whenever existing firms exit either exogenously or endogenously, their z projects become available to potential entrant entrepreneurs such that the mass of projects is always equal to one. New entrants start their operation with zero capital and zero loans. Thus, the measure of entrants $\gamma(z)$ is given by the following:

$$\gamma(z) = \int [(1-\delta)d(K, B_R, z) + \delta]\, \Upsilon(K, B_R, z)\mathbf{d}(K \times B_R).$$

The evolution of the distribution of firms is given by

$$\Upsilon'(K', B_R', z') = \int Q((0,0,z),(K', B_R', z'))\gamma(z)\mathbf{d}z +$$

$$(1-\delta)\int (1 - d(K, B_R, z))Q((K, B_R, z),(K', B_R', z'))\Upsilon(K, B_R, z)\mathbf{d}(K \times B_R \times z), \tag{9}$$

where $Q(\cdot)$ denotes a transition function that maps current states into future states. The distribution of firms the following period includes the set of surviving firms that do not exit exogenously or endogenously. It also includes the new firms that enter after project opportunities from the exiting firms become available.

The *recursive equilibrium* for this economy consists of the policy and value functions of firms, the loan contracts offered by creditors, and the distribution of firms such that (i) given the schedule of loan contracts offered, the policy and value functions of firms satisfy their optimization problem; (ii) loan contracts reflect the firm's default probabilities such that with every contract creditors break even in expected value; (iii) the distribution of firms follows (9) and is consistent with the policy functions of firms and shocks given the initial

[21] If $\beta(1 + r) = 1$, firms strictly prefer to accumulate assets rather than distribute dividends because of the additional benefits of assets in terms of avoiding firm failure. This would generate an excessive supply of loans that would in turn drive down the risk-free rate.

distribution Υ_0.

3.4 Borrowing Limits and Financial Development

Limited enforceability of debt contracts generates endogenous borrowing limits for firms because creditors do not provide loans that will be defaulted on in all future states. These borrowing constraints play a key role in determining optimal debt. Moreover, borrowing limits vary across firms and with financial market development. In particular, weak financial development limits borrowing relative to assets. And this limitation is more severe for small firms than for large firms.

We provide an analytical characterization of these findings by considering the case when firms are heterogeneous with respect to z yet this productivity is constant over the firm's lifetime. In addition, for simplicity we assume that $\delta = 0$. We also impose an assumption on credit costs. This assumption guarantees that firms have an incentive to borrow to the limit every period. It also ensures that the borrowing limit is at least as large as the first best level of capital for all firms.[22]

Assumption 2 Credit costs are such that $\xi \leq \left(\frac{\alpha z}{1+r}\right)^{\frac{1}{1-\alpha}} \frac{1-\alpha}{\alpha}, \ \forall z.$

When productivity is certain and constant over time, firms will either repay or default with probability one on any loan. Thus, there is no equilibrium default, as loans that will be defaulted upon with probability one are not offered. Hence, loan contracts available are offered at the risk-free rate. Let $\overline{B}(z)$ denote the borrowing limit of a firm with productivity z and $\overline{B}_R(z) = (1 + r)(\overline{B}(z) + \xi)$ denote the associated debt repayment limit.

The assets of the firm are equal to the level of capital $K_{fb}(z)$, which is constant over time at the first best level, as its return is equalized in equilibrium to the constant return on bonds. Also the firm borrows to the limit given $\beta(1 + r) < 1$. Hence, the value of a new entrant firm with productivity z is given by

$$V^c(0, 0, z) = [\overline{B}(z) - K_{fb}(z)] + \beta[zK_{fb}(z)^\alpha - K_{fb}(z) - r\overline{B}(z) - (1 + r)\xi]/(1 - \beta).$$

The value of any existing firm with productivity z and debt repayment $\overline{B}_R(z)$ is given by

$$V^c(K_{fb}(z), \overline{B}_R(z), z) = [zK_{fb}(z)^\alpha - K_{fb}(z) - r\overline{B}(z) - (1 + r)\xi]/(1 - \beta).$$

[22] In our general model with varying productivity, financial frictions interact with capital decisions even when debt schedules allow for the first best capital because these limited schedules do not permit maintaining the efficient capital and an increasingly larger loan due after a sequence of bad shocks.

The borrowing limit for a firm with productivity z is the level of debt that makes the contract value equal to the default value, and is given by

$$V^c(K_{fb}(z), \overline{B}_R(z), z) = \theta V^c(0, 0, z).$$

Hence, we derive the debt limit as

$$\overline{B}(z) = \kappa_1 K_{fb}(z) - \kappa_2 \xi,$$

where both κ_1 and κ_2 are positive.[23] More productive and larger firms (bigger z) have looser borrowing limits than small firms, independent of the degree of financial market development. Also, stronger financial market development (lower ξ) increases the loan availability for all firms, independent of productivity.

Furthermore, the maximum loan relative to capital for a firm with productivity z is

$$\frac{\overline{B}(z)}{K_{fb}(z)} = \kappa_1 - \frac{\kappa_2 \xi}{K_{fb}(z)}. \tag{10}$$

The relation between debt limits to assets and size is affected by the economy's financial development or easiness to acquire credit information, ξ. When credit information is free ($\xi = 0$), all firms face the same borrowing limits relative to assets. This is because the problem is homogeneous with respect to z. When credit costs are large ($\xi > 0$), small firms are constrained in their borrowing relative to large firms because the credit cost increases default risk disproportionately for them. Moreover, the disadvantage of small firms relative to large firms becomes more pronounced as ξ increases. The following proposition summarizes this finding.

Proposition 1. *In the case without uncertainty, $\delta = 0$, and under assumptions 1 and 2, the relation between debt limits to assets and firm size is decreasing in the degree of financial development:*

$$\frac{d^2\left(\overline{B}(z)/K(z)\right)}{dK(z)d\xi} > 0.$$

Proof. Direct differentiation of equation (10) delivers the result.

Deriving analytical expressions for debt limits in the case of stochastic productivity is difficult due to lack of analytical solutions for the firm's decision rules of debt and investment.

[23] $\kappa_1 = \frac{[(1+r-\alpha)(1-\theta)+\theta(1-\beta)(1+r)]}{\alpha(r(1-\theta\beta)+\theta(1-\beta))}$ and $\kappa_2 = \frac{\alpha(1+r)(1-\theta\beta)}{\alpha(r(1-\theta\beta)+\theta(1-\beta))}$.

However, all the results regarding borrowing limits, sizes, and financial market development carry through when we solve numerically the model for the more general case with uncertainty.

4 Model Quantitative Implications

We now assess quantitatively how default risk shapes firms' financing and growth. We calibrate the model to two representative countries, Bulgaria and the United Kingdom. The important parameters that capture financial frictions are calibrated to obtain the financing patterns observed in the cross section of firms in each country. We then find that default risk can quantitatively account for the relation of firm size with growth found in each country. Improving financial markets in the model reduces the difference in both growth rates and leverage ratios of small versus large firms, which makes the model fully consistent with the empirical evidence. The distinct behavior of entrant, incumbent and defaulting firms in the model as financial development varies also mirrors the findings in the data. Better financial markets also increase significantly the output of small firms.

4.1 Calibration

We calibrate the model twice to match Bulgarian and British data in 2004 and 2005. The following parameters are chosen independently of the model equilibrium. The interest rate r is set to 4% per annum for Bulgaria and 2% per annum for the UK, which corresponds to the real interest rates in these countries from *International Financial Statistics*.[24] The decreasing returns parameter α is chosen to be 0.90, following Atkeson and Kehoe (2005). The probability of re-accessing credit markets after default θ is set to 0.10 following Chatterjee et al. (2007) so that the average number of years that defaulters are excluded from credit markets equals 10 years.

We assume that firms' idiosyncratic productivity consists of a permanent component μ_z and an i.i.d. component ε such that the productivity for firm i equals $z_t^i = \mu_z^i \cdot \varepsilon_t^i$. To make the distribution of firms in our model tractable, we assume that μ_z can take five values $\mu_z \in \{\mu_z^1, \mu_z^2, \mu_z^3, \mu_z^4, \mu_z^5\}$ and that ε_t can take two values $\{\varepsilon_L, \varepsilon_H\}$. Each μ_z is assumed to have equal mass. Without loss of generality, we assume that transitory shocks have a mean of one, and thus the low shock ε_L and its probability p_L are sufficient to capture the transitory idiosyncratic shock process.

[24] The real interest rate is constructed as the difference between the annual nominal lending rate and the inflation rate.

We jointly calibrate $\{\mu_z^1, \mu_z^2, \mu_z^3, \mu_z^4, \mu_z^5, \varepsilon_L, p_L, \beta, \xi, \delta\}$ to best match the following 11 moments in the data: the median asset levels and mean leverage ratios of five asset quintiles, and the average real sales growth rate from 2004 to 2005 in each country. In the model, all these parameters affect all the target moments in a non-linear fashion. Nevertheless, each parameter impacts more some particular moments. Specifically, a larger β or a smaller δ increases the degree of precautionary savings and hence decreases the leverage ratios of all firms. A larger ξ mainly increases borrowing costs of small firms, and thus lowers the leverage ratio of the smaller firms; the smaller the firm the larger the impact of ξ. The stochastic shock structure mainly affects firm size and growth. The μ_z determine the asset quintile levels, and the volatility of the transitory shocks determines the average growth rate. Table 7 summarizes all the parameter values in the calibration.

Table 7: Benchmark Parameter Values

	Parameter	Bulgaria	United Kingdom
Interest rate	r	0.04	0.02
Re-entry prob.	θ	0.10	0.10
Technology	α	0.90	0.90
Permanent prod.	$\mu_z^1, ..., \mu_z^5$	1.3, 1.4, 1.6, 1.8, 2.5	1.3, 1.5, 1.7, 2.1, 3.1
Temporary prod.	$\varepsilon_L, \varepsilon_H$	0.12, 1.08	0.36, 1.11
	p_L	0.08	0.14
Exogenous exit	δ	0.04	0.06
Credit cost	ξ	0.07	0.0
Discount factor	β	0.95	0.97

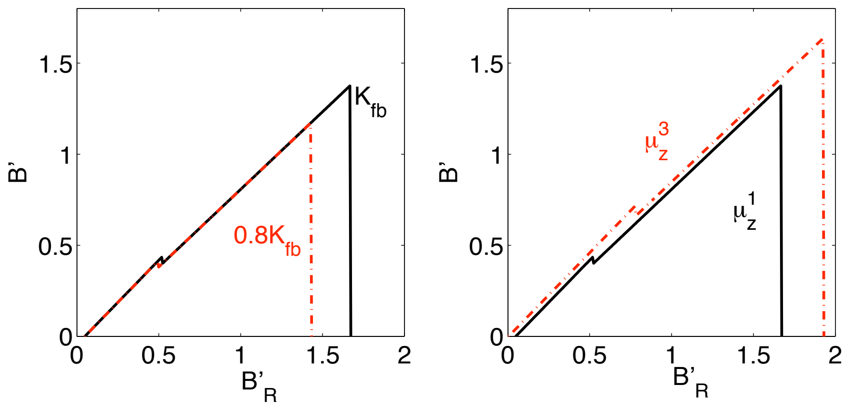
The calibrated ξ parameter for Bulgaria equals 0.07, which corresponds to 0.03% of output for the average firm. The credit costs are higher for the smallest firms and equal 5% of the output of firms in the first asset quintile. The calibrated ξ parameter for the United Kingdom equals zero. The coefficient of variation of the calibrated shock process is 0.26 in the Bulgarian calibration and 0.25 in the British calibration.

4.2 Debt Schedules and Dynamics

Before presenting the quantitative results, it is informative to understand how default risk affects firms' debt schedules and how these restrictions on credit impact firms' choices of debt, capital, and dividends.

Let's start by looking at the equilibrium debt schedules $\Omega(z, K')$ that arise due to default risk. The left panel of Figure 2 shows that debt schedules are more lenient if firms choose larger capital levels. The panel plots the possible pairs (B', B'_R) (relative to the first best

Figure 2: Debt Schedules



capital) for two firms with mean productivity μ_z^1 and capital choice equal to 100% and 80% of the first best level. The slope of the schedule equals $(1-\delta)/(1+r)$ when B_R ranges from 0.02 to about 0.5. Due to the fixed cost, very small loans (with B_R ¡0.02) carry effectively infinite interest rate and thus firms never choose such loans. For larger B_R values, the firm defaults in the low shock the following period; the slope of the schedule in this range is $(1-p_L)(1-\delta)/(1+r)$. For even larger B_R values, the firm defaults with probability one, and thus these contracts are not offered in equilibrium. Default risk not only increases the effective interest rate paid on loans, but also generates borrowing limits. Importantly, these limits are increasing in the capital choice of the firm. As shown in the figure, the maximum transfer B' that the firm can get is 1.7 if the firm chooses the first-best capital or 1.4 if capital is 20% lower.[25]

Debt schedules are also more lenient for firms with higher productivity. The right panel of Figure 2 plots the possible pairs (B', B'_R) (relative to their corresponding first best capital levels) for two firms with mean productivity μ_z^1 and μ_z^3 when capital K' equals $K_{fb}(\mu_z^i)$.[26] As in Proposition 1 for the deterministic case, firms with higher productivity default higher levels of debt relative to their capital. Hence, they can borrow relatively larger loans at both risk free rates and risky rates.

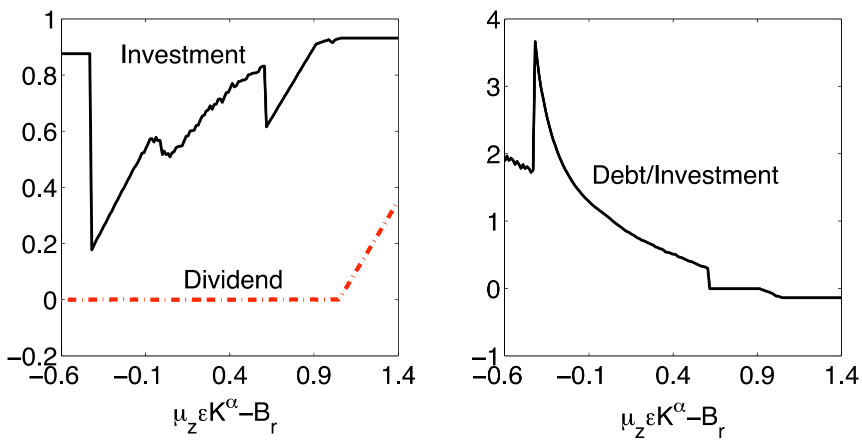
The limitations on debt contracts affect the way firms respond to shocks. When experiencing sequences of bad shocks, firms tend to reduce their scale to maintain non-negative dividends and increase their debt financing, climbing up their debt schedules. When experiencing a good shock, these inefficient firms expand their scale and reduce their debt, sliding down their debt schedules. These dynamics imply that firms with the same perma-

[25]Note that if the ϵ shock were continuous, the schedule would have a smooth inverted U-shape

[26]Given that $K_{fb}(\mu_z^3)/K_{fb}(\mu_z^1)$ is about 10, the difference in the absolute borrowing schedules for these two firms is very large.

nent productivity display different sizes that depend on the history of shocks. Across these firms, inefficiently small firms tend to have higher growth rates and higher leverage ratios. To illustrate these dynamics, consider the decision rules for a firm with median permanent productivity μ_z^3 in the Bulgarian calibration shown in Figure 3.

Figure 3: Policy Rules



Optimal policies depend on the permanent component of productivity and a single endogenous state variable: *cash on hand*, which equals output minus debt repayment, $\mu_z \varepsilon K^\alpha - B_R$. Cash on hand encodes the information regarding the firm's history of transitory shocks and it is low when firms have a low productivity shock, large debt due, and small capital. In Figure 3 we plot the optimal capital choice, dividends, and debt relative to the capital choice as a function of cash on hand. We report capital, dividends, and cash on hand relative to the first best capital for this firm.

With large cash on hand, the firm chooses a large capital stock, distributes dividends, and holds a low level of debt. The low debt level is due to a precautionary motive, similarly as in standard precautionary savings models (Aiyagari 1994 and Huggett 1993). With uncertainty, the firm may not find it optimal to exhaust its borrowing opportunities because large debt increases default risk and hence borrowing costs. In addition, large debt increases the volatility of cash on hand which increases the likelihood of a binding nonnegative dividend condition next period. Thus, the firm has incentives to reduce its debt to this low level whenever possible under good transitory shocks.

With intermediate levels of cash on hand, the firm stops paying dividends, and tends to increase debt and decrease investment. The nonmonotonicity of the capital choice around the income level of 0.6 is due to the fixed credit cost. The firm prefers zero debt to small debt because the fixed credit cost makes the effective interest rates of small debt prohibitively high. Although larger capital choices allow firms to face more lenient debt schedules, large capital choices might also require firms to choose larger loans, such that dividends remain non-negative. The non-monotonicities in the capital choice around the zero income level reflect this trade-off. Overall we find that as the firm's cash on hand decreases, the choice of capital is lower to prevent debt from increasing too rapidly. All else equal, smaller loans are beneficial to avoid future default, which is costly because the expected value of keeping the project is large.

With low levels of cash on hand, the firm has very large debt to repay and finds it no longer optimal to avoid default. In anticipation of default under the low shock the following period, the firm chooses high debt and adjusts investment to a more appropriate scale for the high shock only. In general, the option to default introduces a convexity in the objective function when cash on hand is sufficiently small. As firms' incentives to take risk rise, default becomes more and more likely and the choice of investment is higher.[27]

Firm size depends on its permanent productivity and also its history of transitory shocks. A firm is small either because it has a low level of permanent productivity (*unproductive*) or because it has a sequence of bad shocks (*unlucky*). Both unlucky and unproductive small firms tend to have inefficient scales because either they are closer to their borrowing limits or have restrictive debt schedules. These small firms have high marginal product of capital and growth fast when hit with good shocks, as these shocks alleviate their needs of credit and they can expand their size to a more efficient level. However, unproductive small firms tend to have low leverage ratios given their restrictive loan schedules and unlucky small firms tend to have high leverage ratios as a result of the bad shocks.

4.3 Calibration Results

Let's now examine the model's calibration results. We compute and simulate the model twice: one under the Bulgarian calibration and one under the British calibration. In each simulation, we obtain a model economy with 15,000 firms over 500 periods. The model delivers in the long run a cross-sectional distribution of firms, which we use to compute the model's statistics. For each period, we divide the cross section of firms into five asset quintiles. In the model, firm size equals the assets of the firm: capital K, plus savings B_R if $B_R < 0$. We compute for every asset quintile and for the entire distribution of firms, average sales growth rates, leverage ratios, and median asset levels. The model results are reported in Table 8, together with the data statistics. Asset levels throughout the paper are normalized to the mean asset level in the first quintile under the Bulgarian calibration.

The table shows that our calibration is successful in matching the target moments in the

[27]The discrete jump in investment in the default region of Figure 3 is an artifact of our two discretized transitory shocks.

Table 8: Quantitative Model Results

Bulgaria	Data			Model		
Quintile	Assets	Growth	Lev.	Assets	Growth	Lev.
1	1	0.67	0.16	1	0.55	0.26
2	4	0.44	0.31	4	0.48	0.33
3	10	0.45	0.40	9	0.43	0.38
4	30	0.43	0.47	30	0.38	0.36
5	693	0.35	0.48	693	0.44	0.47
Mean	148	0.47	0.36	147	0.46	0.36
U.K.	Data			Model		
Quintile	Assets	Growth	Lev.	Assets	Growth	Lev.
1	1.4	0.19	0.73	1.4	0.20	0.70
2	7.3	0.13	0.62	7.4	0.17	0.65
3	34	0.11	0.57	34	0.12	0.55
4	202	0.09	0.54	205	0.11	0.54
5	12267	0.09	0.53	12259	0.15	0.61
Mean	2502	0.12	0.60	2501	0.15	0.61

data. In particular, the model generates a tight fit in terms of the average sales growth, the median asset level and mean leverage ratio of each asset quintile in each country. The model replicates the upward sloping relation of leverage and size in Bulgaria and downward sloping relation of leverage and size in the UK.

We now evaluate the model implications on the size-growth pattern. The model under both calibrations generates a negative size-growth relation similar to that in the data. Specifically, under the Bulgarian calibration, the model growth rates for the smallest and largest firms are 0.55 and 0.44, respectively, close to the data counterpart of 0.67 and 0.35. Similarly under the British calibration, the growth rate declines from 0.2 for the smallest firms to 0.15 for the largest firms in the model, while it declines from 0.19 to 0.09 in the data.

The firm-specific debt schedules drive these results. In the Bulgarian calibration, the overall results are driven by the variation in debt schedules across firms with different permanent productivity as the majority of firms in asset quintile i have permanent productivity μ_z^i. The unproductive small firms in economies with weak financial markets have disproportionately restrictive debt schedules, which induces these firms to have low leverage ratios and high growth rates. The model under the British calibration is homogeneous across permanent productivity as $\xi = 0$, and thus the overall results are driven by the unlucky firms who have climbed up their debt schedules. These small unlucky firms have higher debt to asset ratios and also higher growth rates, and hence the model can match the size-growth and size-leverage patterns of the British economy.[28]

The model also predicts that large firms are the ones that distribute dividends, which is consistent with U.S. data as documented in Fazzari et al. (1988). In the Bulgarian calibration, 60% of firms in the first asset quintile do not pay any dividends, compared to 49% for firms in the fifth asset quintile. In the UK calibration, 50% of firms in the first asset quintile do not pay any dividends, compared to 49% for firms in the fifth asset quintile. Firms in economies with better financial markets are more likely to pay dividends because of a lower precautionary motive due to larger loan availability.

Our results demonstrate that financial frictions can rationalize quantitatively the majority of the growth differential across firms observed in both Bulgaria and the UK. These exercises are revealing because we use the financial variables of firms to discipline the extent to which the growth-size relation can be attributed to financial imperfections. By parameterizing the model to mirror the debt financing patterns of firms, we find that the model delivers a close fit to the observed the growth-size relation in the data.

4.4 Improving Financial Development

We now use our calibrated model to analyze the consequences of improving the development of financial markets in Bulgaria by reducing the credit cost to the UK level. We compare the model's predictions for the size, growth, and leverage of incumbent and entrant firms across economies to the empirical findings in section 2. We also evaluate how financial development affect the propensity to default for firms with varying size and leverage and compare these model predictions to the data.

Growth, Leverage and Size

Table 9 presents the average size, growth and leverage for all firms grouped by asset quintiles when we lower credit costs in the Bulgarian calibration to the UK level. When financial markets develop, the size-leverage relation becomes negative, and the size-growth relation becomes flatter. In particular, the difference in growth rates between the smallest and largest firms declines from 11% to 5%, and the difference in leverage ratios increases from −21% to 5%. Also, lowering credit costs increases the mean leverage from 36% to 45%, and decreases the mean growth rate from 46% to 40%. All these implications are fully consistent with our empirical findings.

[28]The intrinsic positive comovement of growth rates and leverage ratios present in our model with zero credit costs is similar to the one analyzed by Cooley and Quadrini (2001), Albuquerque and Hopenhayn (2004), and DeMarzo and Fishman (2007).

Table 9: Counterfactual Experiments

	Bulgaria benchmark			UK credit cost		
Quintile	Assets	Growth	Lev.	Assets	Growth	Lev.
1	1	0.55	0.26	0.9	0.45	0.50
2	4	0.48	0.33	3	0.41	0.47
3	9	0.43	0.38	9	0.39	0.44
4	30	0.38	0.36	30	0.35	0.40
5	693	0.44	0.47	700	0.40	0.45
Mean	147	0.46	0.36	149	0.40	0.45

Lowering the credit cost allows firms to survive a longer series of bad shocks by shrinking their size instead of defaulting. This implies a lower aggregate exit rate and smaller asset levels in the first quintile in countries with better financial development.[29] Better financial development also impacts the output of small firms. When the fixed credit cost is lowered from 0.07 to 0, the average output rises by 10% for firms in the first asset quintile. This is because more unlucky firms with high permanent productivities fall into this asset bin and produce larger output under the lower credit cost. Lowering credit costs has a modest positive impact on aggregate output which is driven mainly by the behavior of large firms who choose on average a more efficient size due to the larger loan availability.

An additional implication of the model is that the fraction of zero-leverage firms declines as financial markets improve. A positive fixed credit cost makes small loans extremely expensive, and thus firms prefer to hold zero debt. As credit costs decline, these small loans become attractive and hence firms switch from zero debt to a modest positive debt level. In addition, firms have zero leverage when they hold savings for precautionary reasons. Lower credit costs reduce the incentives to engage in precautionary savings, especially for the unproductive small firms for which the credit cost is relevant. This force also decreases the fraction of zero-leverage firms. We find that these two forces reduce the fraction of zero-leverage firms in the first asset quintile from 82% to 50%, when credit cost becomes zero. This model implication is qualitatively consistent with the data. We compute the fraction of firms with a leverage ratio less than 5% in the smallest asset quintile for our sample countries. This fraction is 62% in Bulgaria and 22% in the UK, and is negatively correlated with the private credit over GDP ratio across countries.

The experiment of lowering credit cost reveals that the differential growth and leverage ratios across firms and economies are mostly driven by financial factors. Nevertheless, other

[29]This model implication is consistent with the data. Using data from the European Comission Report we find that the correlation between the private credit to GDP ratio and the average exit rate is -0.34, and the correlation between credit bureau coverage and exit is -0.36.

country's characteristics such as the productivity structure and the risk free rate have a large impact on the average sales growth and leverage across all firms. The impact of these parameters can be seen by comparing the results from the UK calibration in Table 8 to the ones under the Bulgarian calibration but with UK credit costs in Table 9.

Entrants versus Incumbents

Table 10 reports the model implications on the behavior of entrants relative to incumbents across economies with varying financial market development. We classify firms with age less than or equal to two as entrants, following the classification in the empirical section. We report the mean leverage ratio and the average sales growth rate for each asset quintile and each group of firms. We find that entrants overall have higher sales growth rates and higher leverage ratios than incumbent firms. In less financially developed economies, the relation of size and growth is more negative among entrant than among incumbent firms. For example in the Bulgarian calibration, the difference in the sales growth rate of the small and large firms is 0.70 for entrants, but only 0.10 for incumbents. In the economy with the UK credit costs, the relation of growth and size for entrant firms is flat, because all entrant firms are homogeneous. The model's predictions for the size-growth of entrant versus incumbents across economies with varying financial development is in line with the data: as financial development improves, the difference in sales growth of small and large entrant firms relative to incumbents shrink.

In terms of leverage, our model predicts that in less financially developed economies, the leverage ratio of entrant firms is negatively related to firm size, whereas leverage increases with size for incumbent firms. However, in more financially developed economies the model predicts that entrant firms have a flat size-leverage relation whereas the relation of size-leverage is downward sloping for incumbents. Importantly, the model predictions on leverage across economies with varying financial development are also consistent with the regression analysis: as financial development improves, the difference in leverage of small and large entrant firms relative to incumbent firms shrinks.

Default, Leverage and Size

Our model allows for endogenous firm default. The model predicts that small and highly leveraged firms have a higher probability of default in less financially developed economies. Specifically, in the Bulgarian calibration the probability of default for firms in the first asset quintile is 0.51 compared to 0.34 for firms in the fifth asset quintile. If this economy faces the UK credit cost, the average probability of default equals 0.28 for firms in the first asset quintile, while it is 0.27 for firms in the fifth quintile. Similarly for leverage, when we divide

Table 10: Entrants, Incumbents, Exits and Financial Development

	Bulgarian Benchmark				UK credit cost			
	Sales Growth		Leverage		Sales Growth		Leverage	
Quin	Entry	Incumbent	Entry	Incumbent	Entry	Incumbent	Entry	Incumbent
1	1.37	0.54	1.15	0.26	0.43	0.46	1.06	0.48
2	0.61	0.48	1.01	0.33	0.43	0.42	1.06	0.44
3	0.71	0.43	0.99	0.38	0.43	0.40	1.06	0.41
4	0.70	0.38	0.99	0.36	0.43	0.36	1.06	0.37
5	0.67	0.44	0.99	0.47	0.43	0.41	1.06	0.43
mean	0.81	0.45	1.03	0.36	0.43	0.41	1.06	0.43

firms according to 5 leverage quintiles in both economies, we find that only firms in the fifth leverage quintile have a positive default probability. In the Bulgarian calibration, firms in fifth leverage quintile have an average leverage ratio of 1.46 and an average defaulting probability of 1.88%. When the credit cost equals zero, the largest leverage bin has a higher average leverage ratio of 1.60 and a lower average default probability of 1.37%.[30]

We test these implications using a linear probability model of financial distress on leverage, size and financial development. We pool data over all years from 1999 to 2005 and include in the regression as dependent variables, the firm's leverage ratio and its asset share in its country and year for the measure of size. We also add 2 interaction terms of leverage and size with financial development as well as fixed effects at the country×year level. The financial distress variable equals one if the legal status of the firm is either "in liquidation" or "in bankruptcy".[31] We find the following regression results:

$$\textit{Distress Dummy}_{k,c} = -0.02 + 0.013 Lev_{k,c} - 0.006 Size_{k,c} - 0.01 Lev_{kc} \times FD_c + 0.004 Size_{kc} \times FD_c, \quad (11)$$

where all coefficients except the constant term are significant at the 1% level with clustered standard errors. *FD* is measured with private credit to GDP.

The empirical results show that both higher leverage and smaller size are associated with higher likelihood of financial distress. As financial development improves, the adverse impact of higher leverage and smaller size are mitigated. All these implications are consistent with our model implications.[32]

[30] These implications are consistent with Campbell, Hilscher, and Szilagyi (2006), who find that small firms with high leverage ratios have a higher failure rate among public firms in the United States.

[31] The coverage of firms' legal status varies across countries. The regression includes only the 13 countries that contain this information across all years.

[32] When *FD* is measured with credit bureau coverage, all implications preserve except the coefficient of the interaction between size and FD which might be due to limited coverage on the financial distress measure across countries.

4.5 Sensitivity Analysis

In this section we conduct sensitivity of our results to the fixed credit cost ξ and to the re-entry probability θ. We show that the size-leverage and size-growth relations vary monotonically with the fixed credit cost ξ and that higher θ makes the relations of size with leverage and growth less negative.

The left panel of Table 11 shows the average growth and leverage for firms in each asset quintile as ξ takes values of $\{0.01, 0.02, 0.05\}$, which are three intermediate values between our benchmark values of 0.07 and 0. All the other parameters are the same as those in the Bulgarian benchmark calibration. The table shows that the size-leverage and size-growth relations vary monotonically with the fixed credit cost ξ.

In the model, the reentry probability of entrepreneurs after default, θ, can also control the severity of the financial frictions, in addition to the fixed credit cost ξ. A large θ limits the economy wide availability of credit and hence can proxy for weak financial market development. To highlight the role of the reentry probability, we set the fixed credit cost ξ at 0 and vary θ from 0% to 50% to 80%, while keeping all the other parameter the same as those in the Bulgarian benchmark calibration. The model implications are reported in the right panel of Table 11. The table shows that the size-growth and size-leverage relations become less and less negative as the reentry probability θ increases.

Table 11: Varying Credit Cost and Re-entry Probability

	$\xi = 0.05$		$\xi = 0.02$		$\xi = 0.01$		$\theta = 0$		$\theta = 0.5$		$\theta = 0.8$	
Quin	Gro	Lev	Gro	Lev	Gro	Lev	Gro	Lev	Gro	Lev	Gro	Lev
1	0.55	0.28	0.49	0.36	0.50	0.45	0.45	0.50	0.48	0.40	0.37	0.31
2	0.49	0.37	0.44	0.43	0.46	0.46	0.41	0.47	0.49	0.42	0.37	0.31
3	0.41	0.40	0.43	0.41	0.44	0.45	0.39	0.44	0.46	0.38	0.37	0.31
4	0.39	0.40	0.38	0.41	0.35	0.41	0.35	0.40	0.47	0.39	0.37	0.31
5	0.44	0.47	0.44	0.47	0.44	0.47	0.40	0.45	0.47	0.40	0.37	0.31
Mean	0.46	0.38	0.44	0.42	0.44	0.45	0.40	0.45	0.47	0.40	0.37	0.31

Interpreting financial market development with θ would make the model consistent with the documented cross country size-leverage patterns, but at odds with the documented cross-country size-growth patterns. To understand these results, recall that when the fixed credit cost is zero the model is homogeneous across permanent productivities. In this case, the size-leverage and size-growth relations are solely driven by the history of transitory shocks. Small firms have on average higher growth and leverage because they have climbed up the debt schedules and are inefficient in scale. A rise in θ increases incentives to default because firms' default values relative to their repayment value rise. Hence, entrepreneurs value their projects relatively less and are reluctant to shrink firms' sizes to protect their project. As a result, in the small asset quintiles there is a smaller fraction of unlucky firms with high permanent productivities. Thus, the average leverage ratios and the average growth rates across asset quintiles become similar to those across permanent productivity. It is also worth mentioning that without the fixed cost ξ variation of θ alone cannot produce a positive size-leverage relation observed for many countries.

5 Conclusion

We have studied both empirically and theoretically the growth and debt financing patterns of firms across countries. Using a broad and comprehensive firm-level database from 27 European countries, we documented that small firms grow faster and use less debt financing than large firms. More importantly, as financial development improves, the growth rate of small firms decreases, but the leverage ratio of small firms increases, relative to large firms, especially for young firms. These findings are robust to controlling for age, sector, and country fixed effects. Our empirical analysis provided a new picture of the relation of financial market development with debt financing and growth across firms and countries.

We then developed a quantitative dynamic model of heterogeneous firms where financial development affects firm financing and growth through the availability to credit. By calibrating the degree of financial development to the observed debt financing patterns of firms, we assessed the model implications on firm growth. We found that financial market development is important in explaining quantitatively the difference in growth rates and debt financing across firms and across countries. We also found that in our model small and highly leveraged firms are more likely to default in less financially developed countries. This additional implication of the model is also borne in the cross country data.

A contribution of the paper is to use micro firm-level data in a quantitative model to study the growth and financing patterns in the cross section of firms of multiple countries. A natural next step is to analyze a time dimension by introducing aggregate fluctuations in the model to study the cyclical features of firm dynamics. Moscarini and Postel-Vinay (2009) document that for the United States, the variance in the firm size distribution is procyclical, and the early phases of booms are mainly driven by the expansion of small firms. Our framework can prove useful in analyzing the impact of financial frictions on the cyclical cross-sectional firm dynamics. More generally, we view our quantitative methodology that combines firm-level data with theory as a useful tool for analyzing the interaction of micro decisions with macro implications.

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Empirical Appendix

In this appendix, we first describe in detail the procedure for cleaning the data in the regressions. We then examine the comparability of the country samples. Finally, we present additional robustness of the main empirical regressions.

Data Cleaning Procedure

We now describe the detailed procedures in assembling the cross-country datasets analyzed in the empirical section. In particular, we present step-by-step data cleaning procedures, construction methods of all variables, and data sources for the country-level statistics.

Firm Data

We download the data from the AMADEUS database compiled by Bureau Van Dijk Electronic Publishing for years 1999–2005. We first delete all firms in the financial and government sectors which correspond to NACE codes 65, 66, 67, and 75. For each cross-section regression of year t, we then restrict the sample to firms that report positive assets and nonnegative liabilities in year t and nonnegative sales in year t and $t+1$. We next drop firms with either leverage or sales growth at the top one percentile of the leverage or sales growth distribution of each country. We finally drop the countries with less than one thousand observations after the above cleaning procedures.

We construct all firm-level variables in year t as follows. For each firm, we generate the *leverage* variable by taking the ratio of the firm's total debt to total assets in year t. We generate the *Size* variable by taking logs of the share of the firm asset in its country in year t. We generate the interaction variables by multiplying *Size* by the corresponding variables of interest such as private credit to GDP or credit bureau coverage. We generate the *growth* variable by computing the firm' net sales growth rate from year t to $t+1$. Given the country-level fixed effect in all the regressions, we need not adjust the sales growth with the country-level inflation rate and changes in the exchange rate. We construct dummy variables for age groups. Firms are classified into 7 age groups based on the firm age in terms of years: $[0,5)$, $[5,10)$, $[10,15)$, $[15,20)$, $[20,25)$, $[25,30)$, $[30,\infty)$. We construct the *entry* dummy to be one for firms with age less than or equal to two.

Country Data

The country-level statistics are obtained from various data sources. Private credit to GDP is from the *World Development Indicators* of the World Bank. Credit bureau coverage is from various issues of *Doing Business* of the World Bank. CPI inflation rates are from the *International Financial Statistics* of the International Monetary Fund.

Comparability of Country Samples

This section analyzes the coverage and comparability of the AMADEUS dataset across countries. The European Commission Report 2005 contains information on the distribution of the universe of firms in the business sector for most of the countries in our sample in 2004. They report the percentage of enterprises that have 1–49 employees, 50–250 employees, and over 250 employees. Hence, we compare the fraction of firms for each employment category in our dataset with that in the universe from the report.[33]

Unfortunately, the employment information is not reported for every firm in AMADEUS. On average, about 66% of firms in our cleaned sample report employment statistics in 2004. The lack of employment data can be a severe problem for some countries. For example, only 21% of firms in our cleaned sample of Italy report employment. Moreover, this lack of employment information is the most severe for small firms. Hence, we impute employment measures for firms that do not report employment in AMADEUS. To do this, we run regressions country by country of log(employment) on log(assets) and log(sales). The fit of these

[33]For this comparison, we include only firms in sectors that correspond to the business sectors in the European Commission Report.

regressions is good, with R^2 above 0.6 for all countries.[34] We then impute employment for the firms that do not report it using the estimated coefficients, their assets and sales.

Table 12 reports the firm distribution in AMADEUS and in the universe for countries for which we have data. The table shows that in our sample, the majority of firms are small with less than 49 employees as in the data. The coverage of small firms in our smaple, though large (on average about 80%), is low relative to that in the universe of firms (on average about 98%). In our sample, about 7% of firms have more than 250 employees, but in the universe, less than 1% of firms fall into this category. On the other hand, the coverage in AMADEUS is similar across countries, with 18 out of 27 countries having small firms larger than 80%.

Robustness of the Main Regressions

This section reports the robustness check in Tables 5 and 6 with financial development measured as credit bureau coverage. Table 13 reports the regression coefficients when additional interactions are added to control for alternative explanations. We find that the coefficients of the interaction terms with the entry dummy are almost the same as those in the main regression. The coefficients of the interaction between size and financial development remain the same sign as those in the main regression, though the magnitude is smaller. Thus, the results of the main regressions are maintained under these alternative specifications and this alternative definition of financial development.

We also conduct additional robustness experiments that incorporate three additional interactions: size and investor protection (taken from the Doing Business dataset), size and GDP volatility (measured as standard deviation of GDP growth), and size and stock market capitalization (taken from Beck and Al-Hussainy at The World Bank). The benchmark results of Table 3 are robust to introducing these variables, as shown in the following table.

Table 15 reports the regression coefficients for the previous sample years with financial development measured as credit bureau coverage. Across all the sample years, the size-growth regression generates all the coefficients similar to our main results, in terms of both the magnitude and the statistical significance. Thus, regardless of private credit to GDP or credit bureau coverage, our main results on the cross-country size-growth relation are robust over time. For the size-leverage regression, the coefficients of size and the interaction terms of size with financial development and with entry are similar to those in the main regression across all the years. The only less robust coefficient is the one of the triple interaction term:

[34]Introducing additional controls such as firm age and sector dummies changes the fit of the regressions only marginally.

Table 12: Coverage and Comparability of Country Datasets

	AMADEUS Dataset			EC Universe		
	Small 1-49	Medium 50-250	Large > 250	Small 1-49	Medium 50-250	Large > 250
Belgium	0.897	0.076	0.027		0.009	
Bosnia	0.844	0.130	0.025			
Bulgaria	0.898	0.078	0.024	0.982	0.016	0.002
Croatia	0.972	0.023	0.004			
Czech	0.854	0.114	0.031	0.991	0.008	0.001
Estonia	0.971	0.026	0.004	0.966	0.030	0.004
Finland	0.910	0.062	0.028	0.985	0.012	0.003
France	0.963	0.029	0.008	0.987	0.010	0.003
Germany	0.617	0.212	0.171	0.972	0.023	0.005
Greece	0.892	0.092	0.016			
Hungary	0.991	0.008	0.001			0.001
Iceland	0.989	0.010	0.001			
Ireland	0.523	0.227	0.249			
Italy	0.952	0.039	0.010	0.994	0.005	0.001
Latvia	0.690	0.242	0.068	0.970	0.027	0.003
Lithuania	0.495	0.396	0.108	0.952	0.043	0.005
Netherlands	0.329	0.354	0.317	0.981	0.016	0.003
Poland	0.353	0.453	0.194	0.989	0.009	0.002
Portugal	0.844	0.125	0.031			
Romania	0.973	0.022	0.005	0.971	0.023	0.006
Russia	0.806	0.138	0.056			
Serbia	0.863	0.096	0.042			
Slovakia	0.684	0.239	0.077		0.055	0.014
Spain	0.959	0.034	0.008	0.991	0.008	0.001
Sweden	0.940	0.046	0.014	0.990	0.008	0.002
Ukraine	0.298	0.446	0.256			
United Kingdom	0.738	0.163	0.098	0.978	0.018	0.004
AVERAGE	0.787	0.144	0.069	0.980	0.019	0.004

Table 13: Robustness with Additional Interactions 1

	Leverage		Sales Growth	
FD×Size	-0.002***	-0.002***	0.004***	0.001*
	(0.0002)	(0.0003)	(0.0008)	(0.0008)
Entry×Size		-0.001***		-0.038***
		(0.0000)		(0.0004)
Entry×FD×Size		0.001***		0.034***
		(0.0002)		(0.0008)
Mean Growth×Size	0.005***	0.006***	-0.137***	-0.130***
	(0.0006)	(0.0006)	(0.0024)	(0.0024)
GDP per capita×Size	0.001***	0.001***	0.005***	0.006***
	(0.0002)	(0.0002)	(0.0007)	(0.0007)
Industry×Size	Yes	Yes	Yes	Yes
Adjusted R^2	0.28	0.28	0.07	0.07
Number of observations	2621201	2621201	2621201	2621201

Note: All regressions have a fixed effect at the country×industry×age level. *FD* is measured as credit bureau coverage. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1% and * denotes significant at 10%.

Table 14: Robustness with Additional Interactions 2

	Leverage		Sales Growth	
	Private Credit to GDP	Credit Bureau Coverage	Private Credit to GDP	Credit Bureau Coverage
Size	0.047***	0.039***	-0.124***	-0.043***
	(0.0007)	(0.0006)	(0.0022)	(0.0019)
FD×Size	-0.012***	-0.006***	0.099***	0.022***
	(0.0003)	(0.0003)	(0.0016)	(0.0007)
SMC×Size	-0.004***	-0.003***	0.010***	-0.0051***
	(0.0002)	(0.0002)	(0.0009)	(0.0009)
IP×Size	-0.003***	-0.004***	-0.002***	0.002***
	(0.0002)	(0.0002)	(0.0003)	(0.0003)
GDPVOL×Size	-0.108***	-0.028***	-0.195***	-1.036***
	(0.0041)	(0.0033)	(0.021)	(0.021)

Notes: All regressions have a fixed effect at the country×industry×age level. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1%.

it becomes insignificantly different from zero in 2002 and has an opposite sign in 1999 and 2000.

Table 15: Robustness with Additional Years

	1999	2000	2001	2002	2003
			Leverage		
Size	0.022***	0.023***	0.025***	0.022***	0.017***
	(0.0002)	(0.0002)	(0.0002)	(0.0002)	(0.0001)
FD×*Size*	-0.007***	-0.011***	-0.014***	-0.003***	-0.006***
	(0.0004)	(0.0004)	(0.0004)	(0.0003)	(0.0003)
Entry×*Size*	-0.001***	-0.001***	-0.001***	-0.000***	-0.001***
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Entry×*FD*×*Size*	-0.001*	-0.001**	0.001**	-0.000	0.002***
	(0.0003)	(0.0003)	(0.0003)	(0.0003)	(0.0002)
Adjusted R^2	0.20	0.17	0.17	0.21	0.22
			Sales Growth		
Size	-0.081***	-0.087***	-0.091***	-0.080***	-0.087***
	(0.0017)	(0.0018)	(0.0015)	(0.0015)	(0.0013)
FD×*Size*	0.074***	0.086***	0.085***	0.054***	0.056***
	(0.0018)	(0.0019)	(0.0016)	(0.0012)	(0.0010)
Entry×*Size*	-0.030***	-0.033***	-0.031***	-0.034***	-0.037***
	(0.0007)	(0.0006)	(0.0006)	(0.0005)	(0.0005)
Entry×*FD*×*Size*	0.018***	0.024***	0.024***	0.028***	0.031***
	(0.0013)	(0.0013)	(0.0012)	(0.0010)	(0.0010)
Adjusted R^2	0.08	0.08	0.07	0.07	0.07
Number of observations	1282817	1409644	1595747	1857510	2124784

Note: All regressions have a fixed effect at the country×industry×age level. *FD* is measured as credit bureau coverage. The standard errors reported in parentheses are robust to heteroskedasticity. *** denotes significant at 1%, ** denotes significant at 5%, and * denotes significant at 10%.

Computation Appendix

In this appendix, we provide the computation algorithm. We start by reducing the number of the state variables of the model from three (capital K, debt repayment B_R, and productivity z) to two (cash on hand $x = zk^\alpha - B_R$ and productivity z). This transformation greatly improves the efficiency of computation. We solve the following *transformed problem*:

$$V(x,z) = \max_{d\in\{0,1\}} (1-d)V^c(x,z) + dV^d(z)$$

where

$$V^c(x,z) = \max_{K',B'} \quad x + B' - K' + \beta(1-\delta)EV(x'(z'), z')$$

subject to

$$x + B' - K' \geq 0,$$

$$x'(z') = z'K'^{\alpha} - B'_R \quad \text{for any } z',$$

$$B' + \xi = \frac{B'_R(1-\delta)}{1+r}\left(1 - \int d(x'(z'), z')f(z', z)dz'\right) \quad \text{if } B' > 0,$$

$$B' = \frac{B'_R(1-\delta)}{1+r} \quad \text{if } B' \leq 0,$$

and $V^d(z) = \theta \max\{V^c(0, z), 0\}$.

We compute the transformed problem using value function iterations. We first discretize the state variable x and the decision variables K and B. Note that our calibration has already given the discrete values to both permanent and transitory productivity shocks. We then make an initial guess over the repayment value $V_0^c(x, z)$ and the default choice $d_0(x, z)$. In particular, we assume that $d_0(x, z) = 0$ for any (x, z). The default value $V^d(z)$ is given by $\theta \max\{0, V_0^c(0, z)\}$. We next solve the transformed problem using value function iterations. Consequently, we obtain an updated repayment value and an updated default choice. We continue with the above procedures until both the value function V_n^c and the default decision function d_n converge.