Federal Reserve Bank of Minneapolis
Research Department Staff Report

July 2012

Financial Frictions and Fluctuations in Volatility*

Cristina Arellano

Federal Reserve Bank of Minneapolis and NBER

Yan Bai

University of Rochester

Patrick J. Kehoe

Princeton University, University of Minnesota,
and Federal Reserve Bank of Minneapolis

ABSTRACT

During the recent U.S. financial crisis, the large decline in economic activity and credit was accompanied by a large increase in the dispersion of growth rates across firms. However, even though aggregate labor and output fell sharply during this period, labor productivity did not. These features motivate us to build a model in which increased volatility at the firm level generates a downturn but has little effect on labor productivity. In the model, hiring inputs is risky because financial frictions limit firms' ability to insure against shocks that occur between the time of production and the receipt of revenues. Hence, an increase in idiosyncratic volatility induces firms to reduce their inputs to reduce such risk. We find that our model can generate about 67% of the decline in output of the Great Recession of 2007–2009.

Keywords: Uncertainty shocks, Great Recession, Labor wedge, Firm heterogeneity, Credit constraints, Credit crunch
JEL classification: D52, D53, E23, E24, E32, E44

---

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

The recent U.S. financial crisis has been accompanied by severe contractions in economic activity and credit. At the micro level, the crisis has been accompanied by large increases in the cross-section dispersion of firm growth rates (Bloom et al. 2011). At the macro level, it has been accompanied by a large decline in labor, even though labor productivity has barely fallen. Motivated by these observations, we build a quantitative general equilibrium model with heterogeneous firms and financial frictions in which increases in volatility at the firm level lead to increases in the cross-section dispersion of firm growth rates and decreases in aggregate labor and output in the face of flat labor productivity.

The key idea in the model is that hiring inputs to produce output is a risky endeavor. Firms must hire inputs to produce and take on the financial obligations to pay for them before they receive the revenues from their sales. Because of the separation between the time of production and the receipt of revenues, any idiosyncratic shocks, such as demand shocks, that occur between these times make hiring inputs risky. When financial markets are incomplete, firms have only limited means to insure against such shocks, and hence, they must bear this risk. This risk has real consequences if, when firms cannot meet their financial obligations, they must experience a costly default. In such an environment, an increase in uncertainty arising from an increase in the volatility of idiosyncratic shocks leads firms to pull back on their hiring of inputs.

We quantify our model and ask, can an increase in the volatility of firm-level idiosyncratic shocks that generates the observed increase in the cross-section dispersion in the recent recession lead to a sizeable contraction in aggregate economic activity? We find that the answer is yes. Our model can generate about 67% of the decline in output and 73% of the decline in employment seen in the Great Recession of 2007–2009. More generally, we find that the model generates labor fluctuations that are large relative to those in output, similar to the relationship in the data. Generating such a pattern has been a major goal of the business cycle literature.

Our model has a continuum of heterogeneous firms that produce differentiated products. The demand for these products is subject to idiosyncratic shocks. The volatility of demand shocks is stochastically time varying, and these *volatility* shocks are the only aggregate shocks in the economy. A continuum of identical households supply labor to firms and lend to firms using uncontingent debt through financial intermediaries.

The model has three key ingredients. First, firms hire their inputs—here, labor—and produce before they know their demand. Second, financial markets are incomplete in that firms have access only to state-uncontingent debt and firms default if they cannot pay for their debt. Third, since firms must pay a fixed cost to enter a market, in equilibrium they make positive expected profits in each period that they do not default. The cost of default is the loss of future expected profits.

Given these ingredients, when firms choose their inputs, they face a trade-off between expected return and risk. As firms increase their employment, they increase the expected return conditional on not defaulting, but they also increase the probability of default. For a given variance of idiosyncratic demand shocks, they choose their optimal employment to balance off the increase in expected return against the losses from default. The potential losses from default are an extra cost of increasing labor and thus distort the firm's first-order condition for labor. When the variance of the idiosyncratic shocks increases, at a given level of employment, the probability of default increases, and thus, so does this distortion. In equilibrium, in the face of such an increase in variance, firms become more cautious and decrease employment. At the aggregate level, these firm-level responses imply that when the dispersion of idiosyncratic shocks increases, aggregate output and employment both fall.

The result that firms decrease employment when the variance of demand shocks increases depends critically on our assumption of incomplete financial markets. If firms had access to complete financial markets, there would be no trade-off between expected return and default risk. Thus, an increase in the variance of these shocks would lead to no change in their employment; firms would simply restructure the pattern of payments across states so that they would never default.

In our model, firms optimally time the purchases and sales of uncontingent debt to help meet their financial obligations in the presence of the stochastic revenue stream generated by the demand shocks. In this sense, firms have a precautionary motive to use debt to self-insure. Since firms have only limited means to repay their debt, they face upward-sloping interest rate schedules and a *credit limit*, a maximum amount they can borrow. Firms typically maintain a *buffer stock* of unused credit. By running this buffer stock up and down, firms can partially dampen the fluctuations in labor input in response to volatility shocks.

As in any incomplete market model, the buffer stock can play an important role as a means to absorb idiosyncratic shocks. In particular, if the incentives to build up this buffer stock are sufficiently strong, firms build such a large stock that they greatly dampen fluctuations in labor. A large literature in finance, however, argues that there are substantial costs of maintaining a large buffer stock and that these costs help explain why firms typically have large amounts of debt. In particular, Jensen (1986) argued that, in practice, if firms retain a large amount of their earnings in order to build up a buffer, managers use these funds in ways that benefit their private interests rather than the shareholder interests. Since shareholders understand these incentives, they give the managers incentives to pay out funds immediately rather than retain them. We capture this *Jensen effect* with a simple parameter that controls the weight in the valuation of present payoffs relative to continuation payoffs. This parameter represents the attempts of shareholders to discipline the managers by rewarding them for paying out funds as dividends rather than keeping them as retained earnings.

We consider a quantitative version of the model in which we choose the parameters of the idiosyncratic firm demand shock process so that the model produces the time variation in the cross-section dispersion of the growth rate of sales observed in a panel of Compustat firms. To illustrate the workings of the model, we consider the impulse response after an increase in the volatility of firm-level shocks. When the volatility shock hits, firms pull back on their employment and decrease their debt in order to avoid default and, in equilibrium, leave the default rate constant. This increase in volatility also leads to a tightening in firms' credit limits, which in turn tends to amplify the reduction in employment.

The pattern of both financial responses and employment to this increase in volatility is heterogeneous across firms. Firms with relatively low demand shocks and high existing debt run their buffer stocks down to zero and decrease their employment the most. At the same time, firms with higher levels of demand shocks tend to increase their buffer stocks and decrease their employment less. These heterogeneous financial responses in the aggregate result in both a large increase in the fraction of firms with a zero buffer and an increase in the aggregate buffer stock. In this sense, our model simultaneously produces tighter credit market conditions, in which more firms are constrained in their borrowing, whereas in the aggregate firms are sitting on larger buffers.

In order to illustrate our new mechanism in the simplest context, we have abstracted from some features that economists have argued are quantitatively important in accounting for business cycle fluctuations. These features include intermediate goods and sticky real wages, which we incorporate in two extensions to our benchmark model. In both extensions the output and employment responses to volatility fluctuations are amplified relative to those in the benchmark model.

To understand how these extensions amplify our mechanism, recall that in our model hiring inputs is risky because firms take on financial obligations to pay for them. In the benchmark model, input prices—here, wages—fall when volatility increases, and these general equilibrium effects dampen our mechanism because they make inputs cheaper in times of high volatility. Both extensions make the price of inputs less sensitive to volatility shocks and hence amplify our mechanism.

We have argued that our mechanism is very different from that in the standard real business cycle model: here, output downturns are not accompanied by falls in labor productivity. A recent strain of work in macroeconomics has argued that in classifying alternative mechanisms for business cycle models, another useful step is to compare the labor wedge generated by the model to that in the data. We follow that strain and discuss the implications of our model for the labor wedge. In the benchmark model, the distortion in firms' labor choices contributes to an aggregate labor wedge, but quantitatively, the model can account for about half of the worsening of the labor wedge during the Great Recession. The model extended with sticky real wages, however, can account well for the dynamics of the labor wedge during this period.

We view our model as providing a new mechanism that links increases in firm-level volatility to downturns. To keep the model simple, we have also abstracted from additional forces that would lead it to generate a slow recovery, as has been observed following the Great Recession. In so doing, we follow the spirit of much of the work on the Great Depression, including Cole and Ohanian (2004), that divides the analysis of the downturn and recovery into mechanisms that generate the sharp downturn and mechanisms that generate a slow recovery.

Our work is related to studies that emphasize time-varying volatility. Bloom (2009) and Bloom et al. (2011) show that in the presence of adjustment costs, firms halt their investment and hiring when hit by a high volatility shock. A key difference between our approach and that of Bloom et al. (2011) is that in the latter, the fixed cost frictions manifest themselves as total factor productivity (TFP) shocks and hence as movements in labor productivity. In contrast, in our model and in the data, we focus on an increase in volatility that is accompanied by a large decline in labor without much change in labor productivity.

Christiano, Motto, and Rostagno (2009) and Gilchrist, Sim, and Zakrajsek (2010) also explore the business cycle implications of volatility shocks. Christiano, Motto, and Rostagno (2009) show that, in a dynamic stochastic general equilibrium model with nominal rigidities and financial frictions, volatility shocks to the quality of capital account for a significant portion of the fluctuations in output. Gilchrist, Sim, and Zakrajsek (2010) study the interactions of financial frictions, volatility, and investment. As in Bloom et al. (2011), they find that increases in volatility lead to drops in aggregate TFP and, hence, labor productivity.

Our work is also related to studies on heterogeneous firms and financial frictions. For example, Cooley and Quadrini (2001) develop a model of heterogeneous firms with incomplete financial markets and default risk and explore its implications for the dynamics of firm investment growth and exit. In other work, Cooley, Marimon, and Quadrini (2004) find in a general equilibrium setting that limited enforceability of financial contracts amplifies the effects of technology shocks on output. Finally, several researchers, including Buera, Kaboski, and Shin (2011) and Buera and Shin (2010), have used similar heterogeneous firm models to help account for the relation between financial frictions and the level of development.

In our model, volatility shocks lead credit constraints to endogenously tighten. In a large body of other work, as in Mendoza (2010), productivity shocks lead credit constraints to endogenously tighten. Finally, a recent literature has developed business cycle models in which the exogenous shock is directly to the credit constraint. See, for example, the work of Guerrieri and Lorenzoni (2011), Perri and Quadrini (2011), and Jermann and Quadrini (2012). This approach is complementary to ours.

1. Model

We start by building a dynamic model that incorporates financial frictions and variations in the volatility of shocks at the firm level.

The model has a continuum of identical households, a continuum of heterogeneous intermediate goods firms, final goods firms, and financial intermediaries. The households have preferences over consumption and leisure, and they provide labor services to intermediate goods firms and lend to these firms through the financial intermediaries. The households own all firms and pay lump-sum taxes. The final goods firms are competitive and have a technology that converts intermediate goods into a final good. This technology is subject to idiosyncratic shocks, referred to as *demand* shocks, which affect the relative demand of the final goods firms for different types of intermediate goods. The volatility of these demand shocks is stochastically time varying, and these *volatility* shocks are the only aggregate shocks in the economy.

The monopolistically competitive intermediate goods firms pay a fixed entry cost and then use labor to produce differentiated products. The shocks to the final goods firms' technology make the demand for their good stochastic. The intermediate goods firms can only borrow state-uncontingent debt and, hence, cannot insure away the fluctuations in demand that they face. These firms are allowed to default on their debt, and if they do, they exit the market.

The timing of decisions is as follows. In the beginning of each period, households decide on the amount of labor to supply to intermediate goods firms. The wage rate is determined so that the labor market clears and intermediate goods are produced. Next, the current demand and volatility shocks are realized. Then all other decisions are made simultaneously. The intermediate goods firms set their prices, sell their products to final goods firms, pay their workers, choose whether to repay their existing debts to financial intermediaries, distribute dividends, and choose new borrowing and a plan for employment. The final goods firms buy the intermediate goods and sell their final goods to households and new entrants. Potential new firms decide whether to enter the market and buy some final goods in order to pay their entry costs. Households consume, receive payments on existing funds lent to intermediaries, and lend new funds to intermediaries.

A. Intermediate and Final Goods Firms

Intermediate goods firms produce differentiated products that are subject to idiosyncratic demand shocks z_t that follow a Markov process with transition function $\pi_z(z_t|z_{t-1}, \sigma_{t-1})$, where σ_{t-1} is an aggregate shock to the standard deviation of idiosyncratic demand shocks. The aggregate shock σ_t follows a Markov process with transition function $\pi_\sigma(\sigma_t|\sigma_{t-1})$.

These intermediate goods firms are monopolistically competitive and produce at the beginning of the period, before the idiosyncratic demand shocks and the aggregate shock are realized. The intermediate goods firms have access to one-period debt contracts and enter period t with a level of debt b_t. They then produce output y_t using the technology $y_t = \ell_t^\alpha$, where ℓ_t is the labor input and $\alpha < 1$. After production, demand shocks are realized.

At this stage, the *idiosyncratic state* of a firm is x_t and the *aggregate state* is S_t. The idiosyncratic state $x_t = (\ell_t, b_t, z_t)$ records its labor input used in production, its debt due, and its current idiosyncratic demand shock. The aggregate state S_t includes the beginning-of-period aggregate state S_{bt} together with the current aggregate shock σ_t, so that $S_t = (S_{bt}, \sigma_t)$.

The beginning-of-period aggregate state $S_{bt} = (\sigma_{t-1}, \Upsilon_t, B_t)$ records the beginning-of-period information on aggregate shocks, σ_{t-1}, the measure of firms Υ_t indexed across x_t, and the contingent assets B_t of households. We find it convenient to record the shock z_t in the beginning-of-period aggregate state even though an individual firm's z_t is not realized until the middle of the period. This approach is permissible, since there is a continuum of firms of each type (ℓ_t, b_t) at the beginning of the period, so the fraction of these firms that will experience each level of z_t is known.

Final goods firms buy the products from intermediate goods firms. The final good is used for consumption and to pay the fixed cost of starting a new firm. The final good Y_t is produced from the intermediate goods via the technology

$$(1) \quad Y_t \leq \left(\int z y_t(x)^{\frac{\gamma-1}{\gamma}} d\Upsilon_t(x) \right)^{\frac{\gamma}{\gamma-1}},$$

where $y_t(x)$ denotes the intermediate goods produced by a firm with idiosyncratic state x, $\gamma > 1$ is the elasticity of substitution across goods, and z is an element of the firm state

$x = (\ell, b, z)$. The final goods firms choose the intermediate goods $\{y_t(x)\}$ to solve

$$(2) \quad \max_{\{y_t(x)\}} Y_t - \int_x p_t(x) y_t(x) d\Upsilon_t(x)$$

subject to (1), where $p_t(x)$ is the price of good x relative to the aggregate price index, which is the numeraire of this economy. This problem yields that the demand $y_t(x)$ for any good with idiosyncratic state $x = (\ell, b, z)$ and price $p_t(x)$ is

$$(3) \quad y_t(x) = \left(\frac{z}{p_t(x)}\right)^{\gamma} Y_t,$$

where $Y_t = Y(S_t) = \left(\int z y_t(x)^{\frac{\gamma-1}{\gamma}} d\Upsilon_t(x)\right)^{\frac{\gamma}{\gamma-1}}$.

Let us turn now to the details of the problem faced by intermediate goods firms. These firms have access to one-period debt in the form of discount bonds that are not contingent on either the idiosyncratic or the aggregate shocks. After shocks are realized, each firm decides on the price of its product. Firms also decide on whether to repay or default on their debt, decisions denoted $\phi = 1$ or $\phi = 0$, respectively. Firms that repay continue, whereas firms that default exit.

Firms that continue in period t must choose new debt contracts b_{t+1} and labor input ℓ_{t+1} at the end of period t before the realization of the new shocks that occur at the beginning of period $t+1$. Note that under this timing, when firms are borrowing at the end of period t, they commit to their plan of production ℓ_{t+1} for period $t+1$ and that production actually occurs in period $t+1$.[1] A debt contract pays off b_{t+1} at $t+1$ as long as a firm chooses not

[1] An alternative timing is one without commitment in which firms borrow b_{t+1} at the end of period t and then are free to choose whatever employment level they want at the beginning of period $t+1$, before the shocks are realized. Notice that in both scenarios, no shocks are realized between the end of period $t+1$ and the beginning of period t, so the only difference is the difference in implied commitment. Here, firms prefer the commitment outcomes because under commitment, firms confront debt price schedules that reward them for choosing less aggressive labor schedules. Mechanically, with commitment when firms choose their labor input, they take the derivative of the price schedule for bonds. When firms do not have commitment, at the beginning of period $t+1$, they have already received the proceeds of the bond sales, and when they choose their labor, they take no such derivative. Note that this subtlety arises only because of the possibility of default.

to default at $t+1$ and gives the firm $q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1}$ at t. The price $q(S_t, z_t, \ell_{t+1}, b_{t+1})$ reflects the compensation for the loss in case of default and depends on the current aggregate state S_t, the firm's current idiosyncratic shock z_t, and two decisions of the firm—its labor input ℓ_{t+1} and its new debt (or *borrowing*) level b_{t+1}.

The dividends d_t for a continuing firm are restricted to be nonnegative:

$$(4) \quad d_t = p_t \ell_t^\alpha - w(S_{bt})\ell_t - b_t + q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1} \geq 0.$$

Here, p_t is the price of this firm's product and w_t is the wage.

In our model, firms default only if they are forced to do so because their budget set is empty. For a firm in state (x_t, S_t), the budget set is defined as $\Gamma(x_t, S_t) = \{d_t | \ d_t \geq 0\}$, where d_t is given by (4). Clearly, firms with large enough debt have an empty budget set, which forces them to default. That is, given the bond price schedules $q(S_t, z_t, \ell_{t+1}, b_{t+1})$ for new borrowing b_{t+1}, there is a large enough inherited debt b_t at t such that no new debt contract can deliver nonnegative dividends. For such a configuration, the only option for the firm is to default. Such a firm then exits. We capture this formally by requiring that if $\Gamma(x_t, S_t) = \varnothing$, firms default by setting $\phi(x, S_t) = 0$.

All firms choose prices and produce, even those that default. Defaulting firms that make enough revenues to cover their wage bill—those with $p_t \ell_t^\alpha - w_t \ell_t \geq 0$—pay this wage bill and pay the residual revenues to debt holders. If a defaulting firm has insufficient revenues to cover current wages, so that $p_t \ell_t^\alpha - w_t \ell_t < 0$, then the firm pays out all its revenues to labor, the government pays off the residual wages by levying lump-sum taxes on households, and bondholders receive zero.

Let $V(x_t, S_t)$ denote the value of the firm after demand shocks are realized in period t. For any state (x_t, S_t) such that the budget set is empty, this value is zero. For all other states in which the budget set is nonempty, firms continue their operations: they hire labor ℓ_{t+1} and produce output y_{t+1}, choose new debt b_{t+1}, and pay dividends d_t. The value of such continuing firms equals

$$(5) \quad V(x_t, S_t) = \max_{\{p_t, \ell_{t+1}, b_{t+1}, d_t\}} \kappa d_t + (1-\kappa) \sum_{z_{t+1}, \sigma_{t+1}} \pi_z(z_{t+1}|z_t, \sigma_t) Q\left(\sigma_{t+1}|S_t\right) V(x_{t+1}, S_{t+1})$$

subject to the production technology $y_{t+1} = \ell_{t+1}^{\alpha}$, the demand for their product (3), the non-negative dividend condition (4), and the law of motion for aggregate states that evolves according to

$$(6) \quad S_{t+1} = H(S_t).$$

In the firm's problem (5), the aggregate price function $Q(\sigma_{t+1}|S_t)$ is the state-contingent price of final goods at $t+1$ in units of final goods at t. This problem gives the decision rules for prices $p(x_t, S_t)$, labor $\ell(x_t, S_t)$, new debt $b(x_t, S_t)$, and dividends $d(x_t, S_t)$. Together with the budget set, it also gives the decision rule for repayment $\phi(x_t, S_t)$. Note that since the elasticity of demand γ is larger than 1, continuing and defaulting intermediate goods firms set their prices so that they will sell all of their output.

We think of the parameter κ in (5) as a simple way of capturing the tensions between shareholders and managers discussed by Jensen (1986). The idea is that if firms have large amounts of retained earnings, then managers will often use these funds in ways that benefit their private interests relative to the shareholder interests. Since shareholders understand the incentives of managers to inefficiently use such funds, the shareholders design the contracts of the managers to induce them to pay out funds immediately rather than retain them. In this context, κ stands in for the attempts of the shareholders to discipline the managers by rewarding them for paying out profits as dividends rather than keeping profits as retained earnings.

The parameter κ, controlling the *Jensen effect*, plays an important role in our model. In the model, the combination of the lack of insurance against idiosyncratic shocks and the nonnegative dividend condition restricts the ability of the firm to choose the size of employment so as to maximize expected profits. That restriction gives firms an incentive to build up a large amount of savings, which would allow the firm to self-insure against idiosyncratic shocks. By adjusting κ, we can make it attractive for firms to borrow rather than build up large levels of savings.

Most dynamic models of financial frictions face a similar issue. The financial frictions, by themselves, make internal finance through retained earnings more attractive than external finance. Absent some other force, firms build up their savings and circumvent these frictions. In the literature, the forces used include finite lifetimes (Bernanke, Gertler, and Gilchrist (1999), Gertler and Kiyotaki (2011)), impatient entrepreneurs (Kiyotaki and Moore (1997)), and the tax benefits of debt (Jermann and Quadrini (2012)). For a survey of these forces and the role they play, see Quadrini (2011).

Firms make their choices of labor and debt before they know the realization of their current shocks, but they know that once these shocks occur, they can use new borrowing to cover their wage bill and existing debt obligations. We can think of each firm as having a *credit line* $\bar{B}(S_t, z_t)$, which is the maximum amount of resources it can borrow at the end of a period:

$$(7) \quad \bar{B}(S_t, z_t) = \max_{\ell_{t+1}, b_{t+1}} [q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1}].$$

Each firm also maintains a *buffer stock* of potential funds, defined as

$$(8) \quad \bar{B}(S_t, z_t) - q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1},$$

which is the unused portion of their credit line. In our model, firms find it optimal to maintain a buffer stock in order to guard against the possibility of receiving a very low demand shock and being forced to default on existing obligations. As we shall see, when volatility increases, firms raise their buffer stock because they have a greater incentive to guard against default.

Now consider firm entry. The model has a continuum of potential entering firms every period. To enter, firms have to pay an entry cost ξ in period t and decide on the labor input ℓ^e_{t+1} for the following period. The entry costs are paid by households and give the households the claims to all future dividends of the firm. The idiosyncratic demand shocks of new entrants z_{t+1} are drawn from a distribution with transition function $\pi_e(z_{t+1}|\sigma_t)$. The value function of entrants is given by

$$(9) \quad V^e(S_t) = \max_{\{\ell^e_{t+1}\}} -\kappa\xi + (1-\kappa) \sum_{z_{t+1}, \sigma_{t+1}} \pi_e(z_{t+1}|\sigma_t) Q\left(\sigma_{t+1}|S_t\right) V(\ell_{t+1}, 0, z_{t+1}, S_{t+1})$$

subject to the evolution of the aggregate states. This problem gives project sizes for new entrants $\ell^e_{t+1}(S_t)$. Let $M(S_t)$ denote the measure of new entrants.

B. Financial Intermediaries

Competitive financial intermediaries borrow from households and lend to firms. At time period t, an intermediary borrows from households by selling them a vector of state-contingent bonds $\{B_{t+1}(\sigma_{t+1})\}$ at prices $\{Q(\sigma_{t+1}|S_t)\}$ and lends these funds to firms. We now derive the bond price schedules offered to firms. To do so, we use the fact that competition among financial intermediaries implies that every contract that an intermediary offers earns zero profits.

To develop the expression for the value of a contingent loan to a firm, suppose the current aggregate state is S_t, and imagine that a firm with current idiosyncratic shock z_t and labor input ℓ_{t+1} promises, conditional on not defaulting, to pay the intermediary an amount b_{t+1} at $t+1$. The intermediary realizes that at $t+1$, when the aggregate shock is σ_{t+1} and the idiosyncratic shock is z_{t+1}, if the repayment indicator $\phi(\ell_{t+1}, b_{t+1}, z_{t+1}, S_{t+1})$ is 1, then this firm will repay completely, and if this indicator is zero, then it will partially default by repaying only its operating profits, $\max\{p_{t+1}\ell^{\alpha}_{t+1} - w_{t+1}\ell_{t+1}, 0\}$. The intermediary values these repayments using the price for contingent claims $Q(\sigma_{t+1}|S_t)$ on the funds obtained from households. Hence, the value for such a default-contingent loan is given by

$$(10)\quad q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1} = \sum_{z_{t+1},\sigma_{t+1}} Q\left(\sigma_{t+1}|S_t\right)\pi_z(z_{t+1}|z_t,\sigma_t)\phi(x_{t+1}, S_{t+1})b_{t+1}$$

$$+ \sum_{z_{t+1},\sigma_{t+1}} Q\left(\sigma_{t+1}|S_t\right)\pi_z(z_{t+1}|z_t,\sigma_t)[1-\phi(x_{t+1}, S_{t+1})]\max\{p(x_{t+1}, S_{t+1})\ell^{\alpha}_{t+1} - w(S_{t+1})\ell_{t+1}, 0\},$$

where ℓ_{t+1} is part of the state x_{t+1}.

C. Households

At the beginning of period t, households provide labor L_t to firms. After the aggregate shock σ_t and the idiosyncratic shocks are realized, the households choose their consumption C_t and state-contingent asset holdings $\{B_{t+1}(\sigma_{t+1})\}$, get paid their wages w_t, receive aggregate dividends D_t from their ownership of the intermediate goods firms, and pay a lump-sum tax T_t.

The state of the household is the beginning-of-period state S_{bt}. The recursive problem for households is the following:

$$(11) \quad V^H(S_{bt}) = \max_{L_t} \left\{ \sum_{\sigma_t} \pi_\sigma(\sigma_t|\sigma_{t-1}) \max_{C_t, \{B_{t+1}(\sigma_{t+1})\}} \left[U(C_t, L_t) + \beta V^H(S_{bt+1}) \right] \right\}$$

subject to their budget constraint

$$(12) \quad C_t + \sum_{\sigma_{t+1}} Q(\sigma_{t+1}|S_t) B_{t+1}(\sigma_{t+1}) = w_t(S_{bt}) L_t + B_t(\sigma_t) + D_t(S_t) - T_t(S_t)$$

and the aggregate law of motion for S_t given in (6), where, recall, $S_t = (S_{bt}, \sigma_t)$. The aggregate dividend that households receive each period is the sum of all the dividends from incumbent intermediate goods firms net of the entry costs from all newly entering firms, so that

$$(13) \quad D_t(S_t) = \int d(x, S_t) d\Upsilon_t(x) - M(S_t)\xi.$$

The household's problem (11) gives the decision rule for labor, $L(S_{bt})$, and the decision rules for consumption and bond holdings, $C(S_t)$ and $B(\sigma_{t+1}|S_t)$.

D. Equilibrium

In our model, market clearing in the labor market requires that

$$(14) \quad \int \ell(x, S_{t-1}) d\Upsilon_{t-1}(x) = L(S_{bt}),$$

where $\ell(x_{t-1}, S_{t-1})$ is the labor input demand for period t committed to by the firm with state x_{t-1} at $t-1$ and $L(S_{bt})$ is the labor supplied by the household. Market clearing in the final goods market requires that the total consumption by households plus the total investment by newly entering firms equals the total final good output:

$$(15) \quad C(S_t) + M(S_t)\xi = Y(S_t).$$

The government budget constraint requires that the lump-sum taxes levied on households cover any wages not paid for by the defaulting firms:

$$T_t(S_t) = \int [1 - \phi(x, S_t)] \max\{w(S_{bt})\ell - p(x, S_t)\ell^\alpha, 0\} d\Upsilon_t(x),$$

where ℓ is an element of the firm's state $x = (\ell, b, z)$. Next, bond market clearing requires that the repayments by firms to the intermediaries equal the payments by the intermediaries on the bonds purchased from the households, so that

$$(16) \quad \int \big(\phi(x, S_t)b + [1 - \phi(x, S_t)] \max\{p(x, S_t)\ell^\alpha - w(S_t)\ell, 0\}\big) d\Upsilon_t(x) = B_t(\sigma_t),$$

where b and ℓ are elements of $x = (\ell, b, z)$. Finally, the free entry condition for new intermediate goods firms is that

$$(17) \quad V^e(S_t)M(S_t) = 0.$$

The transition function for the measure of firms is $\Upsilon_{t+1} = H(S_t)$, given by

$$(18) \quad H(x_{t+1}; S_t) = \int \Lambda(x_{t+1}, x|S_t)\Upsilon_t(x)dx + M(S_t)\Lambda^e(x_{t+1}|S_t),$$

where the probability that a firm with some $x = (\ell, b, z)$ transits to $x_{t+1} = (\ell_{t+1}, b_{t+1}, z_{t+1})$ in aggregate state S_t is given by $\Lambda(x_{t+1}, x|S_t) = \pi_z(z_{t+1}|z_t, \sigma_t)$ if $\ell_{t+1} = \ell(x, S_t)$, $b_{t+1} = b(x, S_t)$, and $\phi(x, S_t) = 1$ and zero otherwise. Likewise, the probability that a new entrant has a state equal to $x_{t+1} = (\ell_{t+1}, b_{t+1}, z_{t+1})$ is $\Lambda^e(x_{t+1}|S_t) = \pi_e(z_{t+1}|\sigma_t)$ if $\ell_{t+1} = \ell^e(S_t)$ and $b_{t+1} = 0$ and zero otherwise.

We now define the equilibrium of this economy. Given the initial distribution Υ_0 and an initial aggregate shock σ_0, a recursive equilibrium consists of policy and value functions of intermediate goods firms $\{d(x_t, S_t),\ b(x_t, S_t),\ \ell(x_t, S_t),\ \phi(x_t, S_t),\ V(x_t, S_t)\}$; household policy functions for consumption $C(S_t)$, labor $L(S_{bt})$, and savings $B(\sigma_{t+1}, S_t)$; the wage rate $w(S_{bt})$ and discount bond price $Q(\sigma_{t+1}, S_t)$; bond price schedules $q(S_t, z_t, \ell_{t+1}, b_{t+1})$; the mass of new entrants $M(S_t)$; and the evolution of aggregate states Υ_t governed by the transition function

$H(S_t)$, such that for all t: (i) the policy and value functions of intermediate goods firms satisfy their optimization problem, (ii) household decisions are optimal, (iii) loan contracts break even in expected value, (iv) domestic good, labor, and credit markets clear, (v) the free entry condition holds, and (vi) the evolution of the measure of firms is consistent with the policy functions of firms, households, and shocks.

We turn now to the definition for real output (GDP). In our model, the final goods producer has no value added, and hence this producer is a simple device to aggregate the output of the heterogeneous firms—which we refer to as intermediate goods firms—into a single value. Of course, we can equivalently think of these heterogeneous firms as final goods producers and equation (1) as reflecting agents' preferences over these final goods.

Under either interpretation, in this environment, the natural definition of GDP is the sum of output of these heterogeneous firms in base period prices $p_0(x)$:

$$(19) \quad GDP_t = \int_x p_0(x) y_t(x) d\Upsilon_t(x),$$

where we consider a base period in which $p_0(x) = 1$ for all x. It turns out that in the quantitative exercise, the time series for GDP_t and Y_t are nearly identical.

2. Our Mechanism in a Simple Example

Before we turn to our quantitative analysis, we construct simple examples to illustrate our mechanism in its starkest and most intuitive form. Specifically, we show how, in the presence of financial frictions, fluctuations in the volatility of demand shocks give rise to distortions that generate fluctuations in labor. To do so, we compare the optimal labor choice of firms in two environments: one in which they can fully insure against shocks and one in which they cannot insure at all.

Consider a one-period stripped-down version of our model. Firms begin the period with some debt obligations b. They then choose the amount of labor input ℓ to hire to produce using the technology $y = \ell^\alpha$ before the idiosyncratic demand shock z for this product is realized. These shocks are drawn from a continuous distribution $\pi_z(z)$. Given the demand shock z and the aggregate output Y, firms choose the prices p for their products and sell them.

If a firm has sufficient revenues from these sales, it then pays its wage bill $w\ell$ and debt obligations and receives a continuation value V, here simply modeled as a positive constant. If the firm cannot pay its wage bill and debt, it defaults and receives a continuation value of 0.

Consider, first, what happens when financial markets are complete. Imagine that a firm chooses the state-contingent pattern of repayments $b(z)$ to meet its total debt obligations b and, hence, faces the constraint

$$(20) \quad \int_0^\infty b(z)\pi_z(z)dz = b.$$

The firm chooses labor and state-contingent debt to solve the following problem:

$$\max_{\ell, b(z)} \int_0^\infty [p(z)\ell^\alpha - w\ell - b(z)]\pi_z(z)dz + V$$

subject to (20) and the nonnegative dividend condition

$$(21) \quad p(z)\ell^\alpha - w\ell - b(z) \geq 0,$$

where $p(z) = zY^{1/\gamma}\ell^{-\alpha/\gamma}$ is the price the firm sets to sell all of its output and is derived from (3) and $y = \ell^\alpha$. Assume that the debt b is small enough so that it can be paid for by the profits of the firm. Hence, with complete financial markets, the firm can guarantee positive cash flows in every state in period 1 by using state-contingent debt $b(z)$, and the dividend constraint is not binding.

With complete markets, the firm's optimal labor choice ℓ^* is such that the expected marginal product of labor is a constant markup over the wage

$$(22) \quad Ep(z)\alpha\ell^{*\alpha-1} = \frac{\gamma}{\gamma - 1}w.$$

This first-order condition shows that with complete financial markets, fluctuations in the volatility of the idiosyncratic shock z that does not affect its mean will have no impact on a firm's labor choice, since $p(z)$ is linear in z.

Now consider what happens when financial markets are not complete: the existing debt is state-uncontingent, so firms have no way to insure against demand shocks. Here, firms with large employment have to default and exit when they experience low demand shocks, since cash flow is insufficient to cover the wage bill plus debt repayments. Effectively, the firm chooses its labor input ℓ as well as a cutoff productivity $\hat{z}$ below which it defaults, where for any ℓ, $\hat{z}$ is the lowest z such that $p(z)\ell^{\alpha} \geq w\ell + b$, where $p(z)$ is described above. Thus, the firm solves the following problem:

$$\max_{\ell,\hat{z}} \int_{\hat{z}}^{\infty} [p(z)z\ell^{\alpha} - w\ell - b]\pi_z(z)dz + \int_{\hat{z}}^{\infty} V\pi_z(z)dz$$

subject to $p(\hat{z})\ell^{\alpha} - w\ell - b = 0$. This last condition defines the cutoff productivity $\hat{z}$ below which the firm defaults, because for any $z < \hat{z}$, the firm would have negative cash flow. The larger the level of labor ℓ, the larger the probability of default for the firm.

In this environment, the optimal choice of labor does not simply maximize period 1 profits as it does with complete financial markets. Here, the firm balances the marginal increase in profits from an increase in ℓ with the increased costs arising from a higher probability of default that such an increase entails. The choice of ℓ^* satisfies

$$(23) \quad E(p(z)|z \geq \hat{z})\alpha\ell^{*\alpha-1} = \frac{\gamma}{\gamma-1}\left[w + V\frac{\pi_z(\hat{z})}{1-\Pi_z(\hat{z})}\frac{d\hat{z}}{d\ell^*}\right],$$

where $p(\hat{z})\ell^{*\alpha} - w\ell^* - b = 0$ and $\Pi_z(z)$ is the distribution function associated with the density $\pi_z(z)$.

When financial markets are incomplete and firms face default risk, the choice of ℓ equates the effective marginal product of labor in the states in which the firm is operative to the marginal costs arising from increasing labor, which includes the wage and the loss in future value. Condition (23) illustrates the distortion in the firm's first-order condition arising from default risk that makes the marginal product of labor larger than the wage.

Now, in contrast to what happens in complete financial markets, fluctuations in the volatility of idiosyncratic shocks do affect the choice of labor. Increases in volatility typically increase the hazard rate $\pi_z(\hat{z})/[1 - \Pi_z(\hat{z})]$, which in turn leads to a larger distortion and a smaller labor input. More precisely, in the Appendix, we assume that z is lognormally distributed with $E(z) = 1$ and $var(\log z) = \sigma^2$. We show that if the value of continuation V is sufficiently large, then a mean-preserving spread (an increase in σ) leads to a decrease in labor ℓ. The intuition for this result is that an increase in volatility increases the risk of default; hence, firms have incentives to lower this risk by reducing their labor input.

3. Quantitative Analysis

The quantitative analysis of our benchmark model begins with parameterization. We use the impulse responses to show how an increase in volatility leads to a drop in output and labor. We illustrate the importance of the financial structure and the source of shocks by contrasting our results to those of alternative specifications with complete markets or with productivity shocks. We show that the model can account for many of the patterns of aggregates during the Great Recession. Finally, we discuss the business cycle moments implied by the model.

A. Parameterization

Many of the parameters of preferences and technology are fairly standard, and we choose them to reflect commonly used values. We use features of the time variation in the cross-section distribution of firms in the United States to help inform the choice of some key parameters of the intermediate goods firms.

Consider the setting of some standard parameters. The utility function is assumed to take the form

$$(24) \quad u(c, h) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{h^{1+\nu}}{1+v}.$$

We set $\sigma = 2$, a common estimate in the business cycle literature. We set $\nu = 0.5$, which implies a labor elasticity of 2. This elasticity is in the range of elasticities used in macroeconomic work, as reported by Rogerson and Wallenius (2009). The exponent of the production function α is set to the labor share of 0.70. We choose the elasticity of substitution parameter $\gamma = 7.7$ so as to generate a markup of 15%, which is in the range estimated by Basu and Fernald (1997).

Now consider the parameterization of the Markov processes over idiosyncratic demand shocks and aggregate shocks to volatility. We want the parameterization to allow for an increase in the volatility of the idiosyncratic demand shock z while keeping fixed the mean level of this shock. We choose a discrete process for idiosyncratic shocks that approximates one that is autoregressive in the log of z, namely,

$$\text{(25)} \quad \log z_t = \mu_t + \rho_z \log z_{t-1} + \sigma_{t-1}\varepsilon_t,$$

where the innovations $\varepsilon_t \sim N(0,1)$ are independent across firms. We choose $\mu_t = -\sigma_{t-1}^2/2$ so as to keep the mean level of z (as opposed to its log) across firms unchanged as σ_{t-1} varies. The discrete process for the aggregate shocks approximates the continuous process

$$\log \sigma_t = (1-\rho_\sigma)\log \mu_\sigma + \rho_\sigma \log \sigma_{t-1} + \upsilon_t,$$

where $\upsilon_t \sim N(0,\varphi^2)$.

Our discrete Markov chains have two aggregate shocks and five discrete sets of values for demand shocks for each of the two aggregate shocks. These approximations follow the methods of Tauchen and Hussey (1991). We also want to have an additional demand shock low enough such that when financial markets are incomplete, firms default when this shock occurs. When we add such a shock, firms default only at this additional low demand shock and at none of the other levels of the demand shock. We choose the probability of the additional low demand shock to be 2.5%. With this level, the model reproduces failure rates similar to the mean failures since 2000 reported by Campbell, Hilscher, and Szilagyi (2008). The productivity of new entrants is chosen to be the lowest discrete value of the demand shock in light of estimates by Lee and Mukoyama (2012). Using the Longitudinal Research Database of the U.S. Census Bureau, they report that the average size of new entrants relative to incumbents is 0.6. In our model, the corresponding value is 0.7.

We set the rest of the parameters so that the model reproduces salient features of the microeconomic data on firms. We choose the serial correlation of idiosyncratic shocks to be $\rho_z = 0.7$, which is in line with Foster, Haltiwanger, and Syverson's (2008) estimated value. We choose the rest of the parameters governing the aggregate and idiosyncratic shocks $\mu_\sigma, \varphi, \rho_\sigma$, the Jensen effect parameter κ, and the entry cost ξ to target five moments. Four of these moments use Compustat data and are features of the distribution of firms: the mean, standard deviation, and autocorrelation of the cross-section interquartile range (IQR) of annual sales growth and the mean ratio of liabilities to sales. Annual sales growth is computed using quarterly data from 1970 to 2010 as $(sales_t - sales_{t-4})/0.5(sales_{t-4} + sales_t)$ with sales deflated by the consumer price index (CPI) for firms in the Compustat data set with at least 100 quarters of observations. The fifth moment is the fraction of labor employed by entering firms, which is measured by the U.S. Bureau of Labor Statistics. The resulting parameters from the calibration are $\mu_\sigma = 0.18$, $\varphi = 0.13$, $\rho_\sigma = 0.85$, $\kappa = 0.4$, and $\xi = 1.57$. Table 1 shows that the model generates moments similar to those in the data.

B. Impulse Response to a Volatility Shock

Here we describe the impulse response of the aggregate economy to an increase in volatility. We then use the impulse responses of individual firms in this economy as well as firms' decision rules to provide intuition for the model's mechanism.

To set the initial conditions before the shock, we consider a long enough sequence of realizations in which the volatility shock is at its mean, so that all aggregates do not change from one period to the next. We then use as an initial condition the resulting measure over individual states. Starting from this distribution, we suppose that in period 1 the volatility shock increases by one standard deviation and stays there from then onward. To help interpret the magnitude of the shock, note that we choose the initial IQR of sales growth to be equal to its mean value of 18% and that a one standard deviation shock increases the IQR to 23%.

Impulse Responses for the Aggregate Economy

We start with the model's impulse responses at the aggregate level. In Figure 1A, we plot the impulse responses of the main aggregates: output, labor, and consumption for 10 quarters. In period 1, the *impact* period, output and labor do not change because firms produce before shocks. In the period after the shock hits, output falls about 2.4% and labor falls more than 3.2%. Aggregate output and labor fall because incumbent firms reduce their employment and fewer new firms enter.

The dynamics of consumption differ from those of output and labor. On impact, consumption rises about 0.3% and then declines. Consumption rises on impact because investment in new firms falls more than output. If we continued these impulse responses, we would find that in the long run, consumption follows output and they both level off, about 1.6% and 2.5% lower, whereas labor slowly rises and eventually returns to almost its initial level.

In Figure 1B, we plot the impulse responses of the aggregate debt of firms and the measure of firms. We see that higher volatility leads firms to de-leverage by decreasing their debt. This increase in volatility also leads to fewer new firms.

We turn now to analyzing labor productivity, wages, and interest rates. *Labor productivity* is simply the ratio of GDP to aggregate employment, GDP_t/L_t. From Figure 1C we see that labor productivity increases a modest amount, about 0.9%, after the shock. The overall response, however, is fairly flat compared to the responses of output and labor. Next, we see that the wage falls on the shock's impact, about 1.4%, and continues to fall thereafter. The interest rate falls modestly, by about 0.5% (from 2.1% to 1.6%) on impact and then stays slightly depressed.

Impulse Responses of Individual Firms

To shed light on the mechanisms driving the aggregate responses just discussed, we now turn to the responses for the individual firms. Recall that each firm's state (z, ℓ, b) includes its shock, employment, and debt. We plot the responses for employment and debt for three representative firms that happen to have a sequence of low (z_L) demand realizations, medium (z_M) demand realizations, and high (z_H) demand realizations. Here, we set z_M at the mean level of z and set the levels of z_L and z_H to be one standard deviation below and above z_M. The initial employment and debt states for each of the firms are set to the median levels within each z group. When volatility increases, the mean demand shock stays the same and the standard deviation increases. Thus, after the increase in volatility, we adjust the levels of z_L and z_H so that they continue to be one standard deviation below and above the mean, thus lowering z_L, raising z_H, and leaving z_M unchanged.

In Figures 2A, 2B, and 2C we plot the levels of labor ℓ, debt b, and the buffer stock $\bar{D} - qb'$ defined in (8) for each firm for 10 quarters after the shock. For each firm, the employment and debt levels are plotted relative to their levels in period 0, and the buffer stock is reported relative to the contemporaneous level of output.

Consider the responses for the firm with a medium demand shock, z_M. When volatility increases in period 1, on impact, this firm decreases its employment about 1.2%, decreases its debt 0.5%, and increases its buffer stock 0.3% of its output. The intuition is that at the original employment and debt levels, when volatility increases, firms would be forced into default more often. When firms default, they lose the future stream of positive expected profits. To avoid losing this stream, the firm with the medium demand shock z_M takes the precautionary actions of lowering its employment and debt levels. It does this in order to reduce its financial obligations to workers and debt holders due at the end of the period and thereby reduce default risk. This firm also chooses to build up its buffer stock in order to help ensure that it can remain solvent in the face of a more volatile distribution of z.

In the background, the increase in volatility also shrinks this firm's credit line because higher default risk induces financial intermediaries to restrict lending. This effect further amplifies the desire of firms to reduce their employment and debt and to increase their buffer stock.

After the impact period, the firm starts increasing its employment. The reason is twofold. First, since the firm has built up its buffer, it can better self-insure against the more volatile shocks it now faces, making an increase in employment less risky. Second, as we saw in Figure 1, wages have fallen, and this general equilibrium effect also leads the firm to increase its employment.

Figure 2 also shows responses for firms with z_L and z_H. On impact, the firms with z_L and z_H take the same precautionary actions of contracting their employment and decreasing their debt, as does the firm with z_M. Since the conditional means of z vary for these firms, however, the magnitudes of their responses differ. For example, since the conditional mean of the demand shock for the z_L firm falls as volatility increases, this firm decreases employment more than the other two types of firms. Likewise, since the conditional mean of the demand shock for the z_H firm increases as volatility increases, this firm decreases employment less than the other two types of firms. These differential effects persist beyond the impact period. After the impact period, the employment for firms with z_L remains depressed, whereas the employment of firms with z_H increases.

Now consider the behavior of debt and the buffer stock for the firms with z_L and z_H. Recall that the credit line available to all firms shrinks with the shock. As shown in Figure 2C, the firms with z_L start with a low buffer stock, and when the shock hits, such firms reduce their debt and exhaust their credit line. Firms with z_H have a larger buffer stock before the shock and increase it by reducing their debt when the shock hits.

In Figure 3, we show the responses of the aggregate buffer stock and the fraction of firms with zero buffer. We see that the increase in volatility leads to an increase in the aggregate buffer stock at the same time as it leads to an increase in the fraction of firms with zero buffer. These responses occur simultaneously because while most firms increase their buffer stocks, the higher volatility leads to a fatter tail of low shocks. This fatter tail in turn leads to an increase in the number of firms that experience relatively low shocks. Such firms end up running their buffer down to zero.

Debt Overhang and Liquidity

Our model has a type of *debt overhang*, in that all else equal, highly indebted firms choose smaller labor.

To illustrate this phenomenon, in Figure 4A we plot the decision rules for labor as a function of the inherited debt for a firm with z_M, when the volatility is low and when the volatility is high. Clearly, firms with larger existing debt choose smaller employment. Since firms find it optimal to roll over most of their debt, firms that inherit larger amounts of debt also take on more new debt. Firms that have more new debt obligations find it optimal to reduce the risk of default by decreasing their level of employment to a more conservative level. As Figure 4A shows, when debt is large enough, firms default and exit. Note that high debt is disproportionately disruptive in times of high volatility because the level of debt for which the firm shrinks its employment and defaults is lower with high dispersion.

Our model generates default because of firm problems with *liquidity,* not *solvency.* To see that default is due to liquidity problems, note that default happens when firms cannot roll over their debt even though the firm has a positive value. Figure 4B shows the value of the firm as a function of debt for the two aggregate shocks. Clearly, the higher the debt of a firm the lower its value, and, once the debt reaches a critical size, the firm's value discretely jumps down to zero. At this critical value, the firm is able to borrow just enough to pay off its existing debt. Hence, for slightly higher values of debt, the firm cannot borrow enough and must default. The value function jumps at this critical value because by defaulting, the firm loses a strictly positive discounted stream of expected future profits.

This analysis leads naturally to the question, why is the firm with a positive present value of dividends not able to borrow more? The firm cannot borrow freely at the contingent prices used in the valuation of future dividends. In particular, the firm cannot borrow against future dividends and repay different amounts contingent on its idiosyncratic shocks. Because of this friction in asset markets, the firm cannot pledge resources based only on the expected stream of profits. Hence, it is possible for a firm to be illiquid, in that it cannot borrow, even though it is solvent, in that it has positive value.

C. Impulse Responses in Two Reference Models

To help understand the role of incomplete financial markets and the source of aggregate shocks in generating a downturn, we contrast the aggregate impulse responses in our model—hereafter, the *benchmark* model—with those in two alternative models. One has volatility shocks but complete financial markets. The other has incomplete financial markets but aggregate productivity shocks. We see here that both financial frictions and the source of the shocks—volatility instead of productivity—are critical to our benchmark model's results.

Volatility Shocks and Complete Markets

To get a feel for the importance of financial frictions at the firm level, we compare our model to one with *complete markets*, in which we add to the model state-contingent claims that pay off on the realization of both idiosyncratic and aggregate shocks.

When firms can issue state-contingent claims, their employment choices ℓ are undistorted and solve

$$\sum_{z_{t+1},\sigma_{t+1}} Q\left(\sigma_{t+1}|S_t\right)\pi_z(z_{t+1}|z_t,\sigma_t)p(z_{t+1},S_t)\alpha\ell^{\alpha-1} = \frac{\gamma}{\gamma-1}w_t(S_{bt}),$$

where $p(z_{t+1},S_t) = z_t Y(S_t)^{1/\gamma}\ell^{-\alpha/\gamma}$. The existence of complete markets also eliminates default because firms can structure the state-contingent payoffs such that their budget sets are never empty. By eliminating default, complete markets prevent inefficient liquidations and deliver a constant measure of firms in the long run.

Figure 5A plots the aggregate responses to increased volatility shocks in a complete markets model. The difference in responses between the benchmark model and the complete markets model is striking. Volatility shocks have very minor effects on aggregates in the complete markets economy, in contrast to the benchmark economy. Aggregate output decreases slightly, about 0.2%, aggregate employment is unchanged, and consumption increases slightly.

Productivity Shocks and Incomplete Financial Markets

To understand the importance of our source of aggregate shocks, we also compare the responses of our benchmark model to a model in which we replace the aggregate volatility shocks with aggregate shocks to firms' productivities. Specifically, we assume that intermediate goods firms produce output using $y_t = A_t \ell_t^{\alpha}$, where A_t is common across firms. We find that the source of shocks is critical for our results.

We choose a discrete process for the productivity shocks that approximates one that is autoregressive in the log of A, namely,

$$(26) \quad \log A_t = \mu_A + \rho_A \log A_{t-1} + \varepsilon_{At},$$

where the innovations are $\varepsilon_{At} \sim N(0, \sigma_A^2)$ and $\rho_A = 0.85$ and $\sigma_A = 0.008$.

Figure 5B plots for this *productivity shock* model the aggregate responses to a permanent decrease in productivity of 3%. Even though this model has incomplete financial markets, it produces a larger decline in output than in labor. This pattern, which is a typical problem in standard real business cycle models, contrasts sharply with the pattern in Figure 1A of our benchmark model.

The comparison of the impulse responses of wages and labor productivity in the productivity shock model to the benchmark model illustrates that our mechanism provides a force for labor to vary that works very differently from the standard productivity shock channel. In Figure 5C we see that in a recession driven by productivity shocks, wages and labor productivity both fall sharply. In contrast, in a recession driven by volatility shocks, wages fall sharply but labor productivity does not. Indeed, in such a recession, labor productivity slightly increases on impact and then flattens out.

As we have noted, in the Great Recession of 2007–2009, labor productivity is stable even though output and labor both decline sharply. Such a pattern is inconsistent with a recession driven by productivity shocks, even in a model like ours with incomplete markets.

D. The Great Recession of 2007–2009

So far we have investigated the implications for our model following a one-time shock to demand volatility. Here we ask how much of the movement in aggregates in the recession of 2007–2009 can be accounted for by our model. We show that our model can account for much of this movement.

In this experiment, we let the initial number of firms be that which arises in the limit after a long sequence of volatility levels such that the IQR equals the one at the start of the recession in the fourth quarter of 2007 (2007:4). We then choose a sequence of shocks so that the IQR of sales growth that the model produces is similar to that in the data. In Figure 6A we show the IQR of sales growth in the model and the data. The IQR increased substantially during the recession, from 0.17 to 0.31. We think of this procedure as using the data (and the model) to back out the realized sequence of volatility shocks. Given our initial condition and this sequence of shocks, we simulate the model.

The model generates substantial declines in aggregate output and labor over this period. From Figure 6B, we see that over the period 2007:4 to 2009:3, the model generates a decline in output of 6.5%, whereas in the data output declines 9.7%. From Figure 6C, we see that the dynamics of labor are similar to those of output: the model produces about an 8% decline in labor, whereas in the data labor declines about 10%. At the end of the recession (2009:3), the model predicts a slight increase in employment, whereas in the data employment remains depressed. Mechanically, the model produces this small upturn because as Figure 6A shows, the increase in the IQR tapers off; hence, so do our backed-out volatility shocks.

We summarize the overall contraction in both output and labor as the cumulative decline in these variables during this whole event. Using this measure, we find that the model can explain 67% of the overall contraction of output and 73% of the contraction in labor during the Great Recession.

From Figure 6D, we see that the model produces a fairly flat productivity profile for the recession, whereas in the data productivity falls modestly and then rises modestly. Note that both in the model and in the data, productivity at the end of this event is essentially unchanged from what it was at the beginning of this event even though output has fallen 10% from beginning to end.

The response of productivity in our model helps to contrast the mechanism in our model with that of Bloom et al. (2011). They show that in a model with adjustment costs for capital and labor, high volatility generates a large productivity decline. Both models generate a contraction in aggregate output in response to high volatility, but they do so through different margins. In the environment of Bloom et al. (2011), the contraction in output is accounted for by an endogenous decline in productivity, whereas in our model, the contraction is accounted for mainly by a decline in labor with flat labor productivity.

Here we have focused on the Great Recession of 2007–2009. We have not tried to account for the slow recovery after the end of the recession in 2009. As it stands, our model cannot account for the slow recovery. The reason is twofold. First, in the data, our measure of volatility, IQR of sales growth, falls relatively quickly post-2009. Second, our model has a tight link between volatility and output so that when volatility falls, output recovers. One reason for this tight connection is that agents know exactly when the volatility shifts. A more elaborate stochastic structure on information in which agents receive only noisy signals of the volatility would allow the model to break the tight connection. Another reason is that we have abstracted from other mechanisms, such as adjustment costs in labor, search frictions, and so on, that stretch out the impact of shocks on aggregates. Finally, we have abstracted from other shocks, including policy uncertainty shocks, that Baker, Bloom, and Davis (2012) show actually increase further after the end of the Great Recession.

E. Business Cycle Statistics

So far we have focused our quantitative analysis on the impulse responses to a one-time shock and the implications of our model for the Great Recession. We are also interested in briefly exploring the second-moment implications of our model. To do so, we consider the business cycle statistics that the benchmark model generates. To highlight the importance of financial frictions and volatility shocks, we compare these statistics to those generated by the complete markets version of our model with volatility shocks and by our model with aggregate productivity shocks but constant volatility.

These comparisons reinforce our conclusions from the impulse responses. First, a necessary ingredient for volatility shocks to have a large impact on output and employment is the presence of incomplete financial markets. Second, even with imperfect financial markets, standard productivity shocks do not generate much volatility in labor relative to output, but volatility shocks do.

In examining these statistics, it is important to recall that in our benchmark results, we have purposefully abstracted from other shocks in order to highlight the quantitative importance of volatility shocks; hence, our model should not be thought of as a complete model of the business cycle. To keep in mind the standard ranges for business cycle statistics, we also report some statistics from the U.S. data. Specifically, we use quarterly data from 1970:1 to 2011:2 and log and detrend each series with linear trends.

We report the results of our benchmark model in Table 2, which shows that even though our benchmark model has only volatility shocks, it generates highly volatile business cycles. The model generates a volatility (std) of output of 2.6, which is about 80% of the volatility observed in the data. The model also generates a relative volatility of labor to output that is similar to that in the data (1.27 in the model vs. 1.28 in the data). The model generates a lower relative volatility of consumption to output than is in the data (0.31 vs. 0.83). A partial explanation is that our model includes no adjustment costs on new entrants, so even though the share of total output of these firms is small, their investment easily adjusts so as to smooth consumption.

In our model, financial frictions at the firm level, together with volatility shocks, generate time-varying distortions to firms' labor choices, which, in turn, generate movements in labor. To put these implications in perspective, recall that classic frictionless business cycle models with productivity shocks do not generate the high volatility of labor relative to output observed in the data because in those models, there are no distortions in the labor market.

We now turn to the business cycle implications of the two reference models.

As can be seen from the *complete markets* statistics in Table 2, when financial markets are complete, volatility shocks produce only minor fluctuations in aggregates. The volatility of output is tiny relative to the volatility of output either in our benchmark model or in the data. Moreover, even when measured in relative volatility terms, labor is quite unresponsive to volatility shocks.

The *productivity shocks* model also does poorly. Table 2 reports second moments when the economy has a constant volatility of idiosyncratic demand shocks and is hit by aggregate productivity shocks. We choose the volatility of productivity shocks such that the aggregate fluctuations in output in this exercise are similar to those in the benchmark model. In the productivity shock model, as in standard business cycle models with aggregate productivity shocks, the volatility of labor relative to output is much lower, less than 40% of that observed in the data.

4. Extensions: Amplification and the Labor Wedge

So far we have kept our benchmark model simple by abstracting from some features that economists have argued are important in accounting for business cycle fluctuations. Nakamura and Steinsson (2010), for example, have argued that intermediate goods, which make up more than half of gross output, help amplify the effects of shocks in their model. Others, such as Hall (2005) and Shimer (2012), have argued that real wages in the Great Recession have fallen much less than a model with flexible wages predicts, and that incorporating real wage rigidities also amplifies the effects of shocks. We begin with two extensions to our benchmark model, intermediate goods and sticky real wages, and show that both of these extensions amplify our mechanism.

To understand why both extensions amplify our mechanism, recall that in our model, hiring inputs is risky because firms must take on financial obligations to pay for these inputs before knowing the revenues from their sales. In the benchmark model, input prices—here, wages—fall when volatility increases. These general equilibrium effects dampen our mechanism. The reason is that exactly when the volatility shocks induce firms to cut their labor input for any given wage, the wage itself falls, which, all else equal, induces firms to hire more labor. Hence, the fall in wages tends to offset the pull-back effect of the increased volatility. Both of our extensions make the price of inputs less responsive to volatility and hence diminish these offsetting general equilibrium effects. Overall, these extensions amplify our mechanism relative to the benchmark case.

We also discuss our model's implications for the labor wedge. Our motivation is that economists have argued that examining the properties of this wedge are useful in understanding the performance of alternative explanations for business cycles.

A. Amplification Mechanisms

Here we discuss the two amplification mechanisms for our model: intermediate goods and sticky real wages.

Intermediate Goods

Consider first the alteration in technologies. We replace the production function $y_t = \ell_t^\alpha$ for each intermediate good firm by $y_t = (\ell_t^{1-\theta} m_t^\theta)^\alpha$, where m is the amount of the composite good Y used as an intermediate input for a given firm. This composite good is now interpreted as gross output. Next, in terms of timing, in the benchmark model firms choose their labor input ℓ_{t+1} at the end of period t at the same time that they choose their new debt b_{t+1}. In this extension, firms now choose their intermediate inputs m_{t+1} at the same time as their labor input. The rest of the economy is unchanged.

To illustrate the aggregate implications of this economy, we repeat our experiments for the Great Recession and compute the business cycle statistics. The only new parameter is θ, which we set to 0.52, motivated by the work of Nakamura and Steinsson (2010). The rest of the parameters are the same as those in the benchmark.

We begin by repeating our experiment for the Great Recession for this economy. Specifically, we choose a sequence of shocks so that the IQR of sales growth that the model produces is similar to that in the data. In Figures 7A and 7B, we plot the resulting paths for output and labor for this model. Comparing the paths for output and labor from this model, labeled *Intermediate Goods, Large Jensen Effect*, to those from the benchmark model, labeled *Benchmark*, we see that adding intermediate goods greatly amplifies our mechanism. With intermediate goods the drop in both output and labor is significantly greater than in the data. As Table 2 on business cycle statistics shows, because of this amplification, the volatility of output is essentially double that in the benchmark model.

The intuition for the amplification relative to the benchmark is that with intermediate goods, over half of a firm's inputs are composite intermediate goods (so that the cost function for producing y units of output is proportional to $w^{1-\theta}p^{\theta}$, where $p = 1$). When the volatility increases, for any given percentage fall in the wage, since the price of the composite intermediate goods does not change, the cost of inputs falls by less than half as much.

We think of our first experiment with intermediate goods as simply providing a comparative static result, rather than a quantitative evaluation of the impact of volatility shocks on the Great Recession. To perform a more relevant evaluation, we reparameterize the model so that it produces a similar volatility of GDP as in the benchmark model. We choose to adjust only one parameter, the parameter κ governing the Jensen effect. The reason is that this parameter is both important for our mechanism and is set only with indirect measures. Specifically, we lower κ to a level such that the volatility of output in the model coincides with that in the benchmark. The resulting value of κ is 0.1. Comparing the paths for output and labor from this model, labeled *Intermediate Goods, Small Jensen Effect*, to those from the benchmark model, we see that adding intermediate goods produces larger declines in output, especially in employment, and brings the time paths for the model closer to those in the data. Turning to the business cycle statistics, we see that intermediate goods magnifies the volatility of labor relative to that of output.

Sticky Real Wages

In our second extension, we consider a *sticky real wage model.* To show the effects of such stickiness, we simply posit a version of what Hall (2005) refers to as a *partially smoothed wage* by assuming that

$$(27) \quad w_t = \lambda w + (1 - \lambda) w_t^*,$$

where w_t^* equals the consumer's expected marginal rate of substitution between consumption and leisure at the beginning of period t and w is the average wage in the benchmark economy.

Here, λ measures the amount of stickiness in that when $\lambda = 0$ wages are perfectly flexible, as in the benchmark economy.

For this extension, we repeat our experiments for the Great Recession and compute the business cycle statistics. The only new parameter is λ, which we set to 0.80. To analyze the episode of the Great Recession, we choose the sequence of shocks to volatility so that the sticky wage model reproduces the observed IQR for this period.

In Figure 8A we compare the real wages in data, the benchmark model, and the sticky real wage model for the Great Recession. For the data we follow Shimer (2012) in using the Employment Cost Index as a measure of nominal wages and deflate this index by the Core CPI. From this figure we see that over the period of the Great Recession, real wages in the data drop by about 2%, whereas wages in the benchmark model drop over 8%. In contrast, in the sticky real wage economy, real wages drop about the same as in the data.

In Figures 8B and 8C we see that, relative to the benchmark, sticky real wages amplify the output and labor effects of the increase in volatility. In the business cycle statistics, we see that these sticky wages also make output substantially more volatile.

B. Labor Wedge

A recent strain of work in macroeconomics has argued that in classifying alternative mechanisms for business cycle models, a useful approach is to compare the labor wedge generated by the model to that in the data. (See Shimer 2009 for a survey.) Here we ask, do our volatility shocks show up as labor wedges?

The first issue we need to grapple with is that the aggregate labor wedge has been defined for economies with an aggregate production function. For example, Chari, Kehoe, and McGrattan (2007) define the labor wedge as the ratio of the marginal rate of substitution between consumption and leisure to the marginal product of labor in the aggregate production function. Our economy with heterogeneous firms and imperfect financial markets does not admit an aggregate production function. Nevertheless, we follow the spirit of the work on the labor wedge and define it to be the marginal rate of substitution between consumption and leisure to labor productivity

$$(28)\quad 1-\tau_t^L = -\frac{U_{Lt}}{U_{Ct}} / \frac{GDP_t}{L_t}.$$

Figure 9A shows that during the Great Recession, the labor wedge falls about 12%. Our benchmark model generates about half of this fall. Thus, volatility shocks do indeed generate labor wedges, but the wedge is less than in the Great Recession.

A major source of the discrepancy between the benchmark model's labor wedge and that in the data is coming from the dynamics of consumption. As Figure 9B shows, in the benchmark model consumption is roughly constant throughout this time period, whereas in the data, consumption falls about 7%.

From Figure 9A we also see that in the model with sticky real wages, the labor wedge in the model is similar to that in the data. From Figure 9B we see that this improvement is due in large part to the behavior of consumption.

Some economists, such as Gali, Gertler, and Lopez-Salido (2007), have argued that an instructive approach is to decompose this wedge as the product of a firm-side wedge $1-\tau_t^f = (w_t/F_{lt})$ and a consumer-side wedge $1-\tau_t^c = (-U_{lt}/U_{ct})/w_t$. In Figures 9C and 9D we plot this decomposition (in logs). We see that in the data, the worsening of the labor wedge is largely driven by the consumer-side wedge. In the benchmark model, in contrast, the worsening of the labor wedge is driven by the firm-side wedge, and in fact the consumer-side wedge improves. Figures 9C and 9D also show the consumer- and firm-side wedge in the real sticky wage model. In this model, the dynamics of the consumer- and firm-side wedges are closer to that in the data. Of course, for a variety of well-known reasons, including, for example, the nature of the long-term relationship between employees and employers, measured wages in the data may not correspond to their theoretical counterparts. Hence, decomposing the labor wedge into a consumer-side labor wedge and a firm-side labor wedge is controversial.

5. Conclusion

We have developed a model in which fluctuations in the volatility of idiosyncratic demand shocks lead to quantitatively sizeable downturns in output and employment. In the model, as in the recent recession, we observe a large increase in the cross-section dispersion of growth rates by firms and a large decline in labor but relatively flat labor productivity. Hence, we think of the model as a promising parable for the Great Recession of 2007–2009.

References

Baker, Scott, Nicholas Bloom, and Steven J. Davis. 2012. "Measuring Economic Policy Uncertainty." Stanford University Working Paper.

Basu, Susanto, and Fernald, John G. 1997. "Returns to Scale in U.S. Production: Estimates and Implications." *Journal of Political Economy* 105(2): 249–283.

Bernanke, Ben S., Mark L. Gertler, and Simon Gilchrist. 1999. "The Financial Accelerator in a Quantitative Business Cycle Framework." In *Handbook of Macroeconomics*, ed. J. B. Taylor and M. Woodford, vol. 1, chap. 21: 1341–1393. Amsterdam: New Holland.

Bloom, Nicholas. 2009. "The Impact of Uncertainty Shocks." *Econometrica* 77(3): 623–686.

Bloom, Nicholas, Max Floetotto, Itay Saporta-Eksten, Nir Jaimovich, and Stephen Terry. 2011. "Really Uncertain Business Cycles." Stanford University Working Paper.

Buera, Francisco J. and Yongseok Shin. 2010. "Financial Frictions and the Persistence of History: A Quantitative Exploration." NBER Working Paper 16400.

Buera, Francisco J., Joseph P. Kaboski, and Yongseok Shin. 2011. "Finance and Development: A Tale of Two Sectors." *American Economic Review* 101(5): 1964–2002.

Campbell, John Y., Jens Hilscher, and Jan Szilagyi. 2008. "In Search of Distress Risk." *Journal of Finance* 63(6): 2899–2939.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2007. "Business Cycle Accounting." *Econometrica* 75(3): 781–836.

Christiano, Lawrence, Roberto Motto, and Massimo Rostagno. 2009. "Financial Factors in Business Cycles." Northwestern University Working Paper.

Cole, Harold L. and Lee E. Ohanian 2004. "New Deal Policies and the Persistence of the Great Depression: A General Equilibrium Analysis." *Journal of Political Economy* 112, No. 4 pp. 779-816.

Cooley, Thomas F., and Vincenzo Quadrini. 2001. "Financial Markets and Firm Dynamics." *American Economic Review* 91(5): 1286–1310.

Cooley, Thomas, Ramon Marimon, and Vincenzo Quadrini. 2004. "Aggregate Consequences of Limited Contract Enforceability." *Journal of Political Economy* 112(4): 817–847.

Foster, Lucia, John Haltiwanger, and Chad Syverson. 2008. "Reallocation, Firm

Turnover, and Efficiency: Selection on Productivity or Profitability? *American Economic Review* 98(1): 394–425.

Gali, Jordi, Mark Gertler, and J. David Lopez-Salido. 2007. "Markups, Gaps, and the Welfare Costs of Business Cycle Fluctuations." The Review of Economics and Statistics, 89 (1), 44-59.

Gertler, Mark and Nobuhiro Kiyotaki. 2011. "Financial Intermediation and Credit Policy in Business Cycle Analysis." In *Handbook of Monetary Economics*, ed. B. M. Friedman and M. Woodford, vol. 3A, chap. 11: 547–599. Amsterdam: North-Holland.

Gilchrist, Simon, Jae Sim, and Egon Zakrajsek. 2010. "Uncertainty, Financial Frictions, and Investment Dynamics." Working paper, Boston University.

Guerrieri, Veronica and Guido Lorenzoni. 2011. "Credit Crises, Precautionary Savings and the Liquidity Trap." NBER Working Paper 17583.

Hall, Robert E. 2005. "Employment Fluctuations with Equilibrium Wage Stickiness" American Economic Review 95 (1): 50–65.

Jensen, Michael C. 1986. "The Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers." *American Economic Review* 76(2): 323–330.

Jermann, Urban, and Vincenzo Quadrini. 2012. "Macroeconomic Effects of Financial Shocks." *American Economic Review* 102(1): 238–271.

Jones, Charles. 2011. "Misallocation, Economic Growth, and Input-Output Economics," NBER Working Paper.

Kiyotaki, Nobuhiro and John Moore. 1997. "Credit Cycles." *Journal of Political Economy* 105(2): 211–248.

Lee, Yoonsoo, and Toshihiko Mukoyama. 2012. "Entry, Exit, and Plant-level Dynamics over the Business Cycle." University of Virginia Working Paper.

Mendoza, Enrique G. 2010. "Sudden Stops, Financial Crises, and Leverage." *American Economic Review* 100(5): 1941–1966.

Nakamura, Emi, and Jon Steinsson. 2010. "Monetary Non-neutrality in a Multisector Menu Cost Model" *Quarterly Journal of Economics* 125 (3):961–1013.

Perri, Fabrizio and Vincenzo Quadrini. 2011. "International Recessions." NBER Working Paper No. 17201.

Rogerson, Richard and Wallenius, Johanna. 2009. "Micro and Macro Elasticities in a Life Cycle Model with Taxes." *Journal of Economic Theory* 144(6): 2277–2292.

Quadrini, Vincenzo. 2011. "Financial Frictions in Macroeconomic Fluctuations." *Economic Quarterly* 97(3): 209–254. Federal Reserve Bank of Richmond.

Shimer, Robert. 2009. "Convergence in Macroeconomics: The Labor Wedge." American Economic Journal: Macroeconomics 1 (1): 280-297.

Shimer, Robert. 2012. "Wage Rigidities and Jobless Recoveries." University of Chicago Working Paper.

Tauchen, George and Robert Hussey. 1991. "Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models." *Econometrica* 59(2): 371–396.

6. Appendix

To illustrate the effects of increasing volatility on the labor choice of firms in the simple example of Section 2, we consider the case where $\ln(z)$ follows a normal distribution $N(\mu, \sigma^2)$. We assume that $b = 0$ and write the condition (23) using the definitions for the price $p(z) = zY^{1/\gamma}\ell^{-\alpha/\gamma}$, the threshold $p(\hat{z})\ell^\alpha - w\ell = 0$ as

$$(29)\quad E\left[z \geq \frac{w}{A}\ell^{1-\theta}\right] A\theta\ell^{\theta-1} - w = \frac{(1-\theta)wV}{A\ell^\theta}\frac{\pi_z\left(\frac{w}{A}\ell^{1-\theta}\right)}{(1-\Pi_z(\frac{w}{A}\ell^{1-\theta}))},$$

where $\theta = \alpha\left(\frac{\gamma-1}{\gamma}\right)$ and $A = Y^{1/\gamma}$. We convert the distributions to standard normals, use the fact that $E(z) = e^{\mu+\sigma^2/2}$, and write condition (29) as

$$(30)\quad e^{\mu+\sigma^2/2}\Phi\left(\frac{\mu+\sigma^2-\ln\left(\frac{w}{A}\ell^{1-\theta}\right)}{\sigma}\right) A\theta\ell^{\theta-1} - w = \frac{(1-\theta)wV}{A\ell^\theta}h\left(\frac{\ln\left(\frac{w}{A}\ell^{1-\theta}\right)-\mu}{\sigma}\right),$$

where Φ and h are cdf and hazard for the standard normal distribution.

We want to consider the effects of a mean-preserving spread of the distribution. To do so, we set $E(z) = 1$, which implies that $\mu = -\sigma^2/2$. The first-order condition (FOC) (30) becomes

$$(31)\quad \Phi\left(\frac{\sigma^2/2-\ln\left(\frac{w}{A}\ell^{1-\theta}\right)}{\sigma}\right) A\theta\ell^{\theta-1} - w = \frac{(1-\theta)wV}{A\ell^\theta}h\left(\frac{\ln\left(\frac{w}{A}\ell^{1-\theta}\right)+\sigma^2/2}{\sigma}\right).$$

To evaluate how the labor ℓ changes with volatility σ, we totally differentiate condition (31) and get an expression for $d\ell/d\sigma$:

$$(32)\quad d\ell/d\sigma = \frac{\frac{(1-\theta)wV}{A\ell^\theta}h_\sigma(.) - \Phi_\sigma(.)\theta\ell^{\theta-1}}{[\Phi_\ell(.)A\theta\ell^{\theta-1} + \Phi(.)A\theta(\theta-1)\ell^{\theta-2}] + \theta\frac{(1-\theta)wV}{A\ell^{\theta+1}}h(.) - \frac{(1-\theta)wV}{A\ell^\theta}h_\ell(.)}.$$

Using the FOC (31) for the bottom of equation (32) and after some simplification, the comparative static $d\ell/d\sigma$ equals

$$(33)\quad d\ell/d\sigma = \frac{\frac{(1-\theta)wV}{A\ell^\theta}h'(x)\frac{dx}{d\sigma} - \theta\ell^{\theta-1}\phi(y)\frac{dy}{d\sigma}}{\phi(y)\frac{dy}{d\ell}A\theta\ell^{\theta-1} + \Phi(y)A\ell^{\theta-2}(\theta(2\theta-1)) - \frac{\theta}{\ell}w - \frac{(1-\alpha)wV}{\ell^\alpha}h'(x)\frac{dx}{d\ell}},$$

where $x = \frac{\ln\left(\frac{w}{A}\ell^{1-\theta}\right)+\sigma^2/2}{\sigma}$ and $y = \frac{\sigma^2/2-\ln\left(\frac{w}{A}\ell^{1-\theta}\right)}{\sigma}$. Note that $\frac{dy}{d\ell} < 0$, $\frac{dx}{d\ell} > 0$, and recall that $h'(x) > 0$ for a standard normal. Sufficient conditions for $d\ell/d\sigma < 0$ are that $\theta < 1/2$ and that the partial derivatives satisfy $\frac{dy}{d\sigma} < 0$ and $\frac{dx}{d\sigma} > 0$. With these sufficient conditions, the bottom of equation (33) is negative and the top is positive.

Below we show that $\frac{dy}{d\sigma} < 0$ and $\frac{dx}{d\sigma} > 0$ are satisfied when the continuation value is high enough such that the implied default probability is less than 1/2.

Assumption 1 $\theta < 1/2$ and $\{V, \sigma\}$ satisfy

(34) $$V \geq \frac{A\left\{\Phi(\sigma)\exp(\sigma^2/2)\theta - 1\right\}}{h(0)\left(\exp(\sigma^2/2)w\right)^{\theta/(1-\theta)}(1-\theta)}$$

for given A and w.

Lemma 1. $\ln\left(\frac{w}{A}\ell^{*1-\theta}\right) \leq -\sigma^2/2$ *under assumption 1*

Condition (34) with equality is precisely the FOC condition when $\ln\left(\frac{w}{A}\ell^{*1-\theta}\right) = -\sigma^2/2$. As V increases, (34) becomes a strict inequality. If the labor ℓ^* decreases with higher V, then $\ln\left(\frac{w}{A}\ell^{*1-\theta}\right) \leq -\sigma^2/2$ under assumption 1. We can show that when $\theta < 1/2$, $d\ell/dV < 0$ by totally differentiating the FOC

(35) $$d\ell/dV = \frac{\frac{(1-\theta)w}{A\ell^{\theta}}h(.)}{\phi(y)\frac{dy}{d\ell}A\theta\ell^{\theta-1} + \Phi(y)A\ell^{\theta-2}(\theta(2\theta-1)) - \frac{\theta}{\ell}w - \frac{(1-\alpha)wV}{\ell^{\alpha}}h'(x)\frac{dx}{d\ell}} < 0.$$

Note that Lemma 1 implies that the liquidation probability $\delta = \Phi\left(\frac{\ln\left(\frac{w}{A}\ell^{*1-\theta}\right)+\sigma^2/2}{\sigma}\right) \leq 1/2$.

Proposition 1. *As volatility increases, labor declines,* $d\ell/d\sigma < 0$, *under Assumption 1.*

Lemma 1 showed that $\ln\left(\frac{w}{A}\ell^{*1-\theta}\right) \leq -\sigma^2/2$. Hence, $dx/d\sigma = -\frac{\ln\left(\frac{w}{A}\ell^{1-\theta}\right)}{\sigma^2} + \frac{1}{2} > 0$ and $dy/d\sigma = \frac{1}{2} + \frac{\ln\left(\frac{w}{A}\ell^{1-\theta}\right)}{\sigma^2} < 0$. These derivatives imply that $d\ell/d\sigma < 0$ in (33).

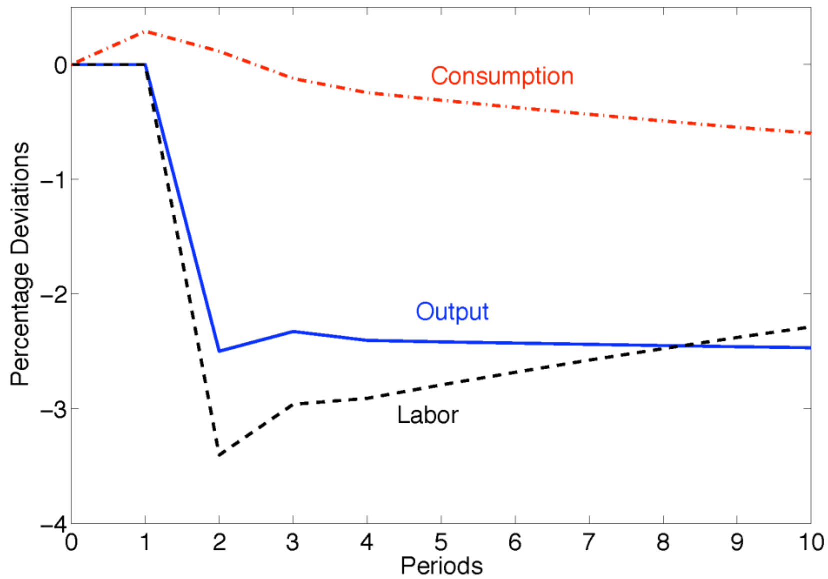
Table 1: Target Moments in Data and Model

Annual Moments	Data	Model
IQR sales growth		
Mean	.18	.18
Std. deviation	4.8	4.7
Autocorrelation	.84	.84
Ratio of liabilities to sales	5.5	5.6
Fraction of labor employed by entry firms	1.8	1.8

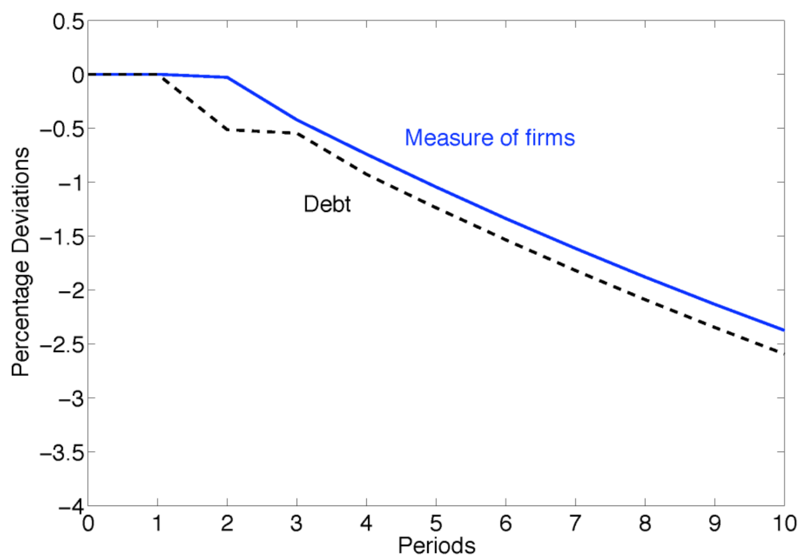
Table 2: Business Cycles Statistics

	$std(\text{GDP})$	$\frac{std(\text{Labor})}{std(\text{GDP})}$	$\frac{std(\text{Consumption})}{std(\text{GDP})}$	$\frac{std(\text{LaborProductivity})}{std(\text{GDP})}$
Data	3.2	1.28	0.83	0.50
Benchmark	2.6	1.27	0.31	0.53
Complete markets	0.2	0.2	1.02	1.02
Productivity shocks	2.0	0.5	0.38	0.96
Intermediate goods Large Jensen effect	5.3	1.8	0.86	1.27
Intermediate goods Small Jensen effect	2.8	1.7	0.83	1.07
Sticky real wages	4.5	1.16	0.64	0.36

Figure 1: Aggregate Impulse Responses to an Increase in Volatility

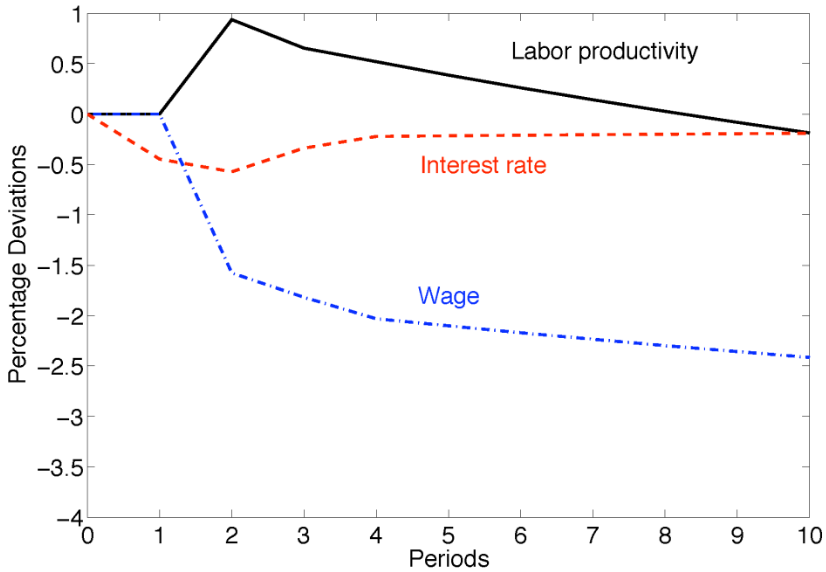


A. Output, Labor, and Consumption



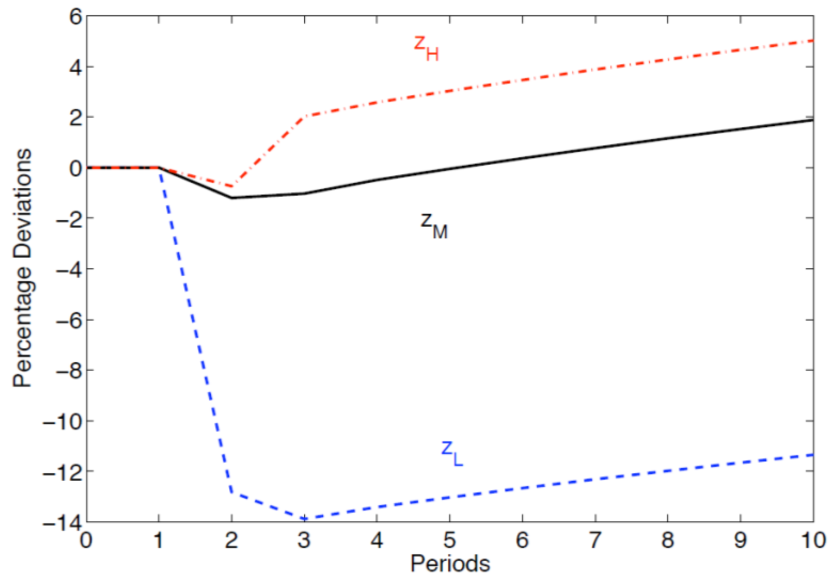
B. Debt and Measure of Firms

Figure 1: Aggregate Impulse Responses to an Increase in Volatility (Cont.)

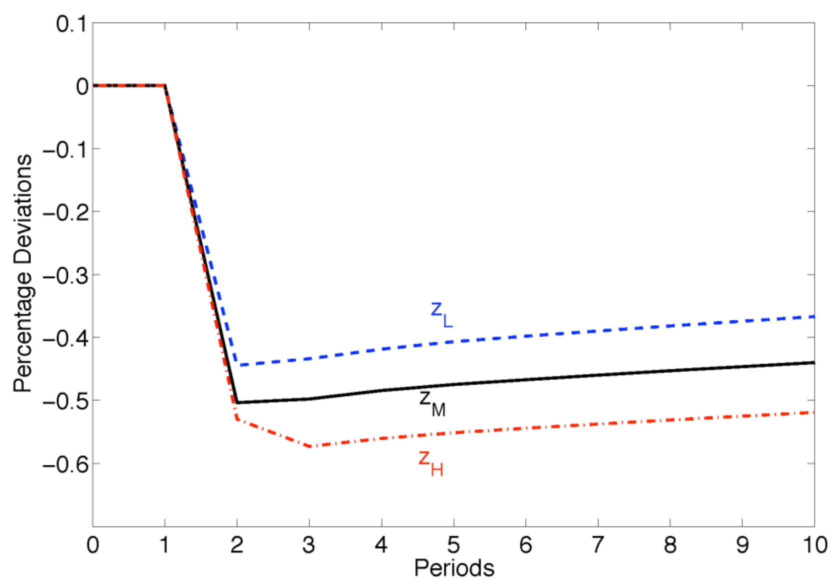


C. Labor Productivity, Wage, and Interest Rate

Figure 2: Firm-Level Impulse Responses to an Increase in Volatility

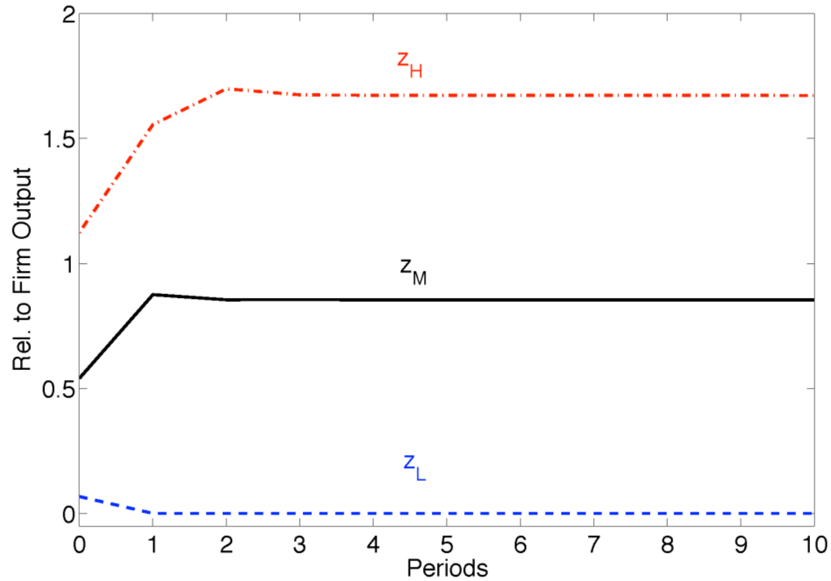


A. Labor



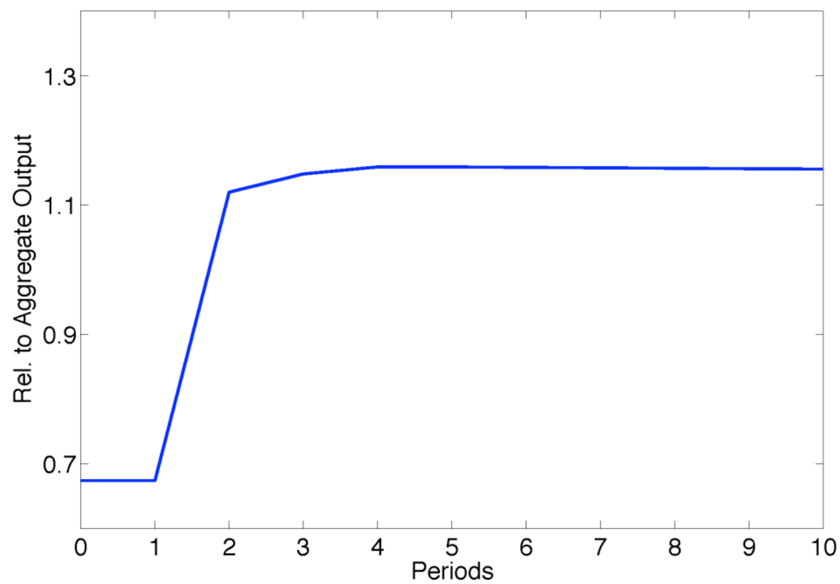
B. Debt

Figure 2: Firm-Level Impulse Responses to an Increase in Volatility (Cont.)

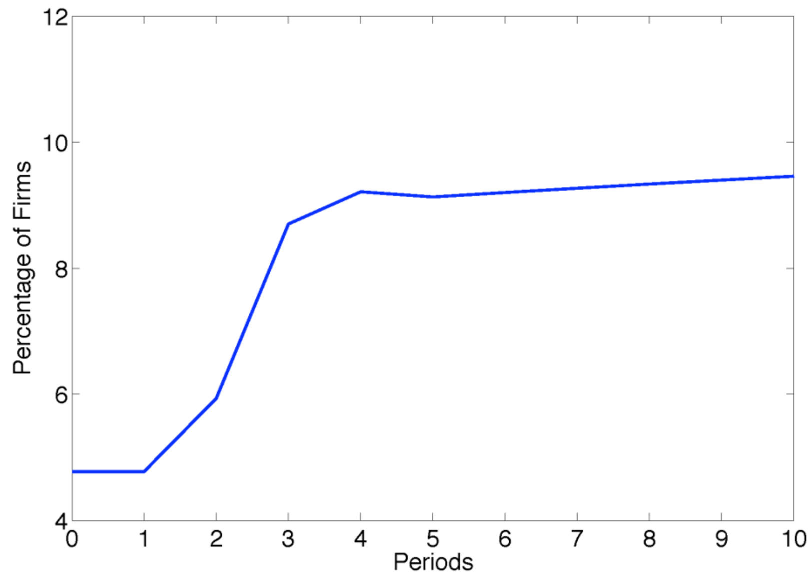


C. Buffer Stock

Figure 3: Aggregate Impulse Response to an Increase in Volatility

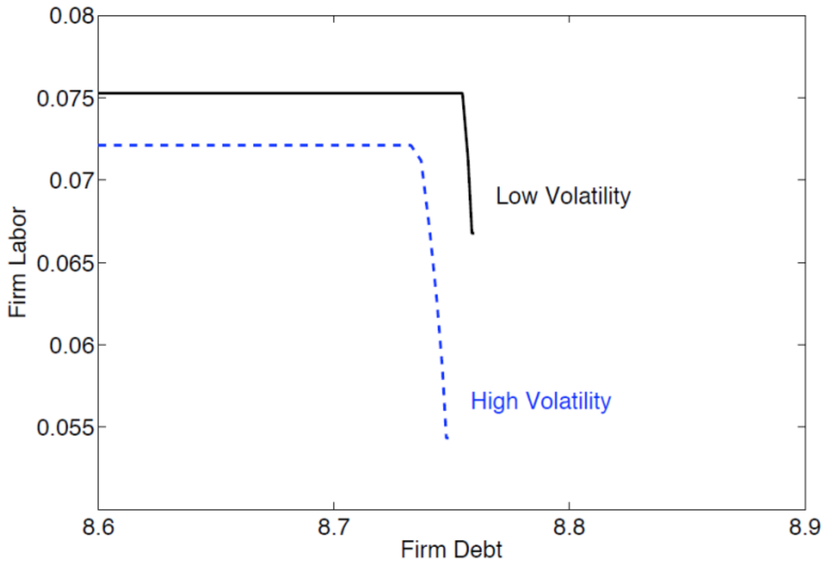


A. Aggregate Buffer

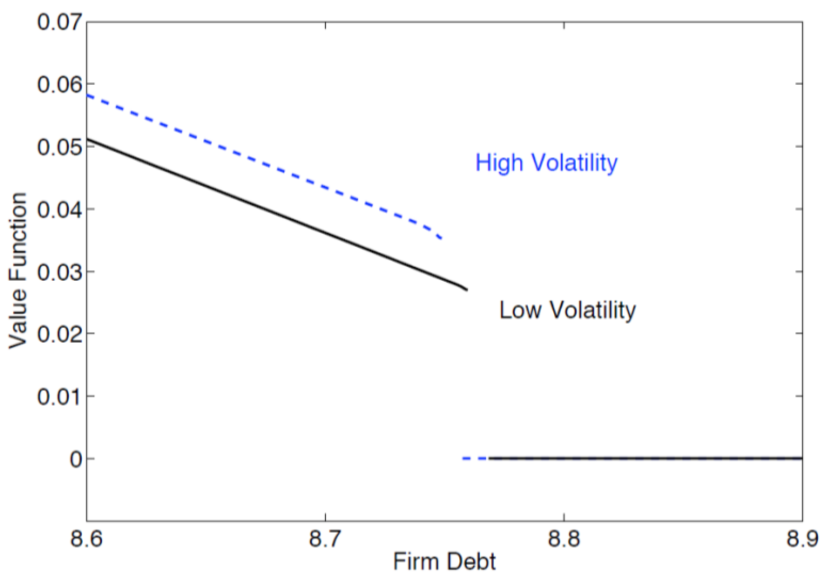


B. Firms with Zero Buffer

Figure 4: Labor Policy Function and Value Function

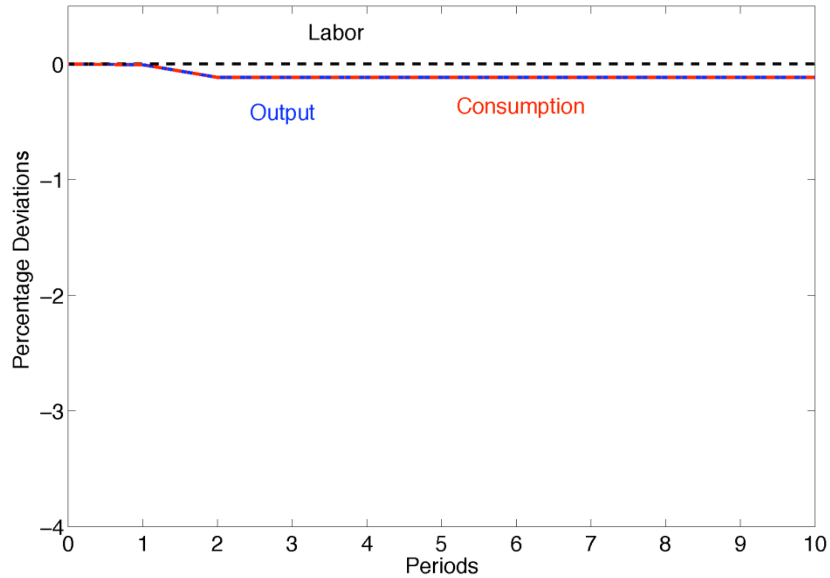


A. Firm Labor Policy as Function of Debt

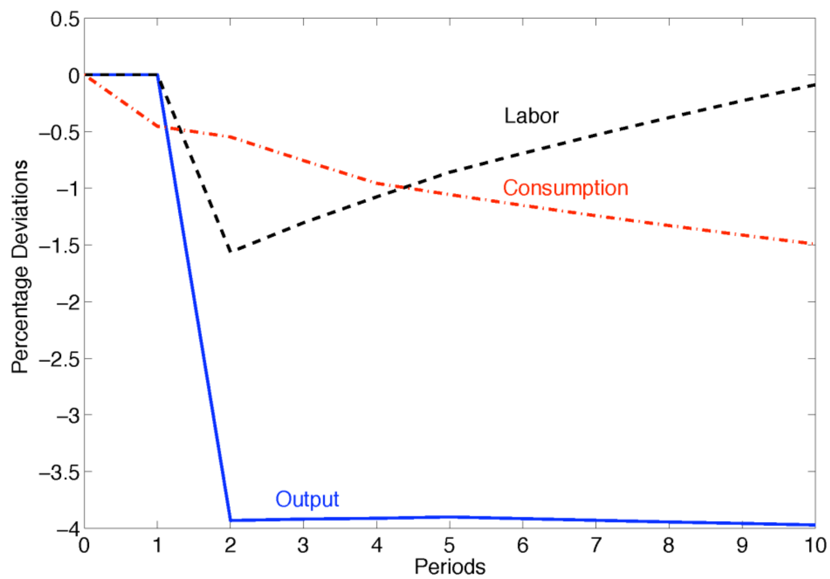


B. Firm Value Function as Function of Debt

Figure 5: Aggregate Impulse Responses in Two Reference Models

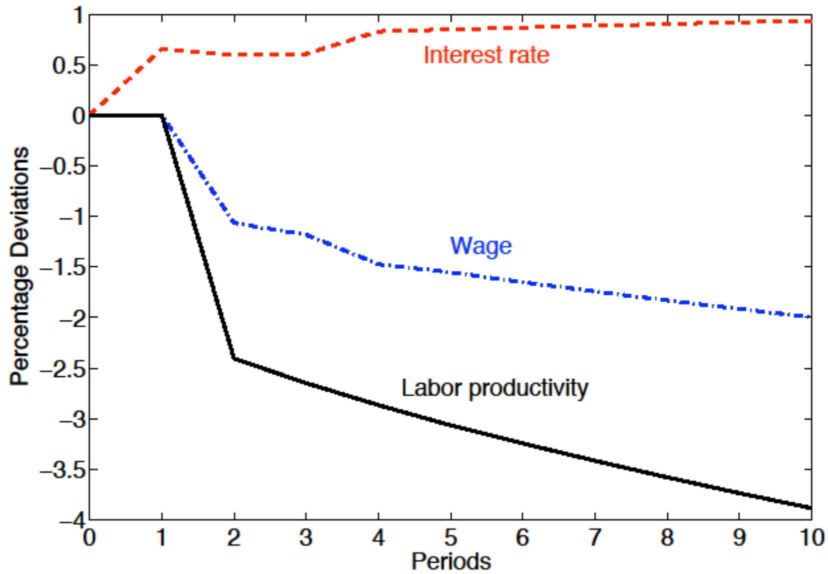


A. Complete Markets Model with Volatility Shocks



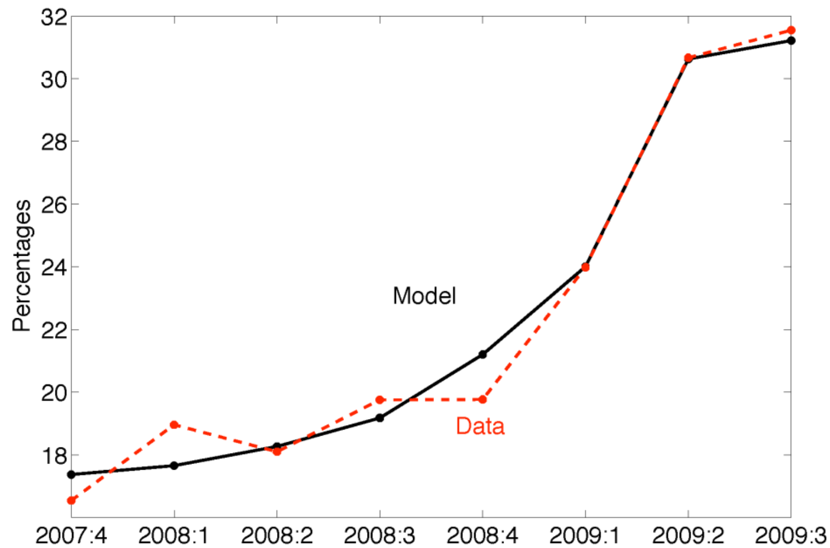
B. Model with Productivity Shocks

Figure 5: Aggregate Impulse Responses in Two Reference Models (Cont.)

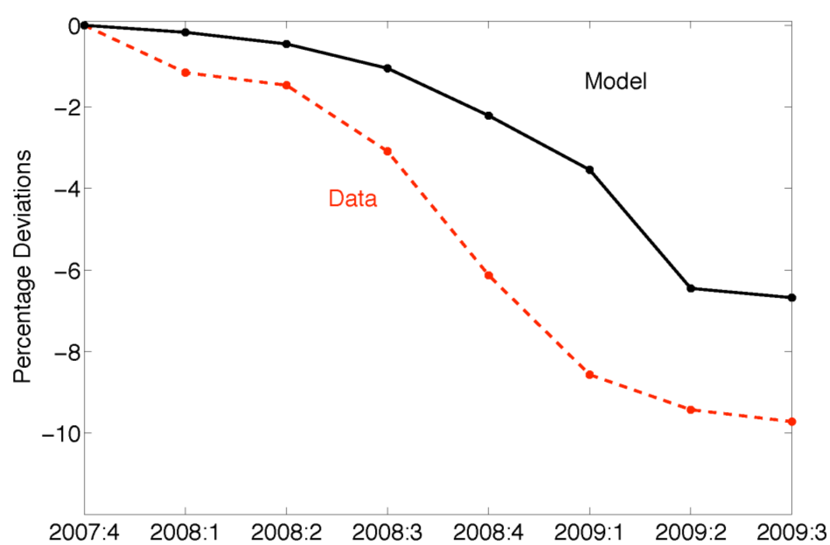


C. Model with Productivity Shocks: Labor Productivity, Wage, and Interest Rate

Figure 6: The Great Recession of 2007–2009

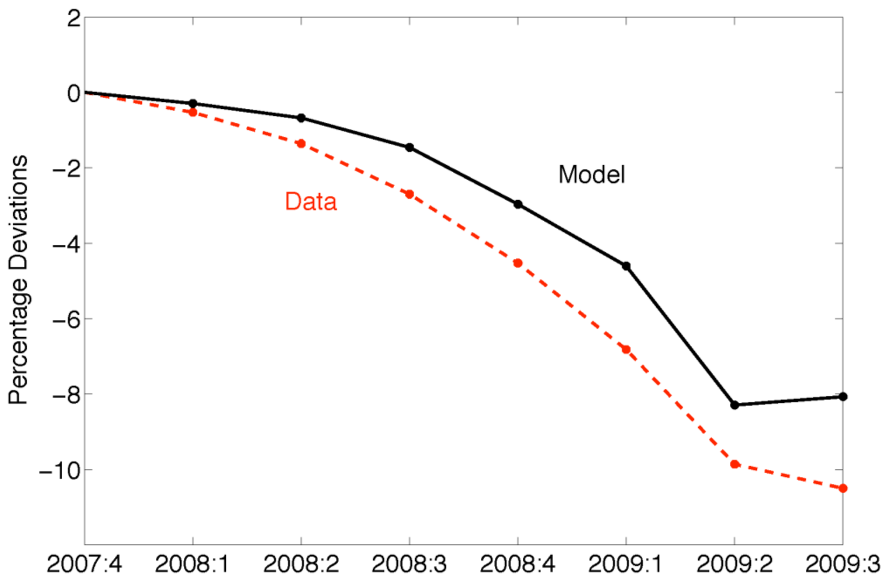


A. Interquartile Range of Sales Growth (IQR)

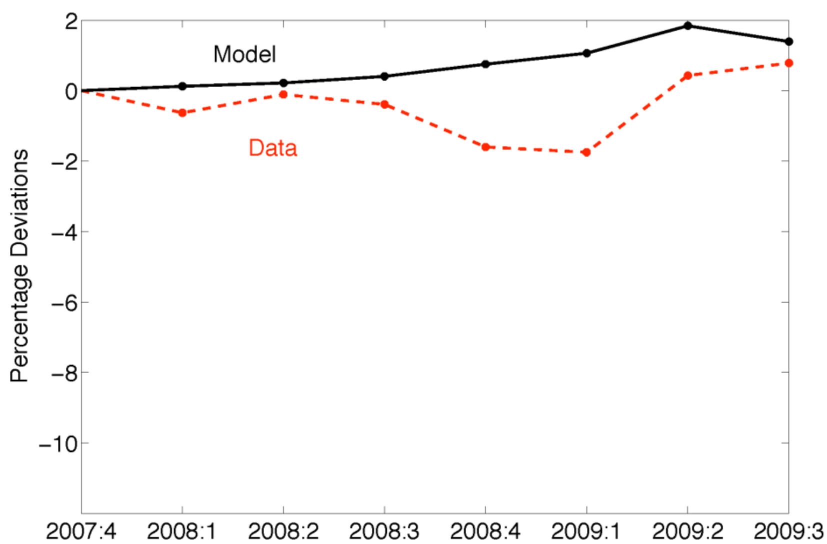


B. Output

Figure 6: The Great Recession of 2007–2009 (Cont.)

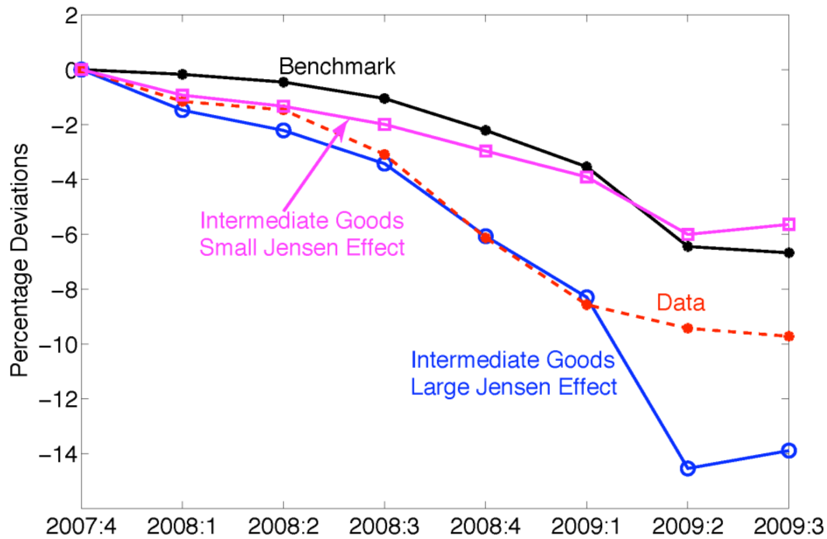


C. Labor

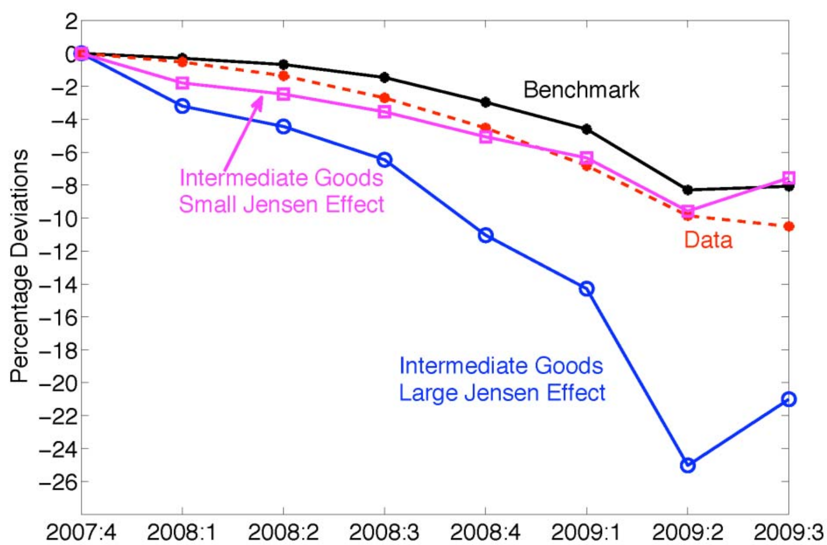


D. Labor Productivity

Figure 7: The Great Recession of 2007–2009: Model with Intermediate Goods

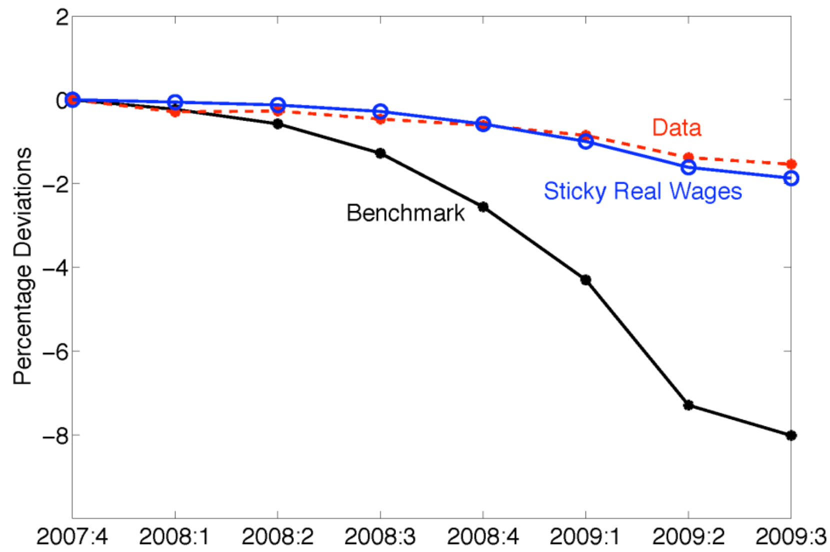


A. Output

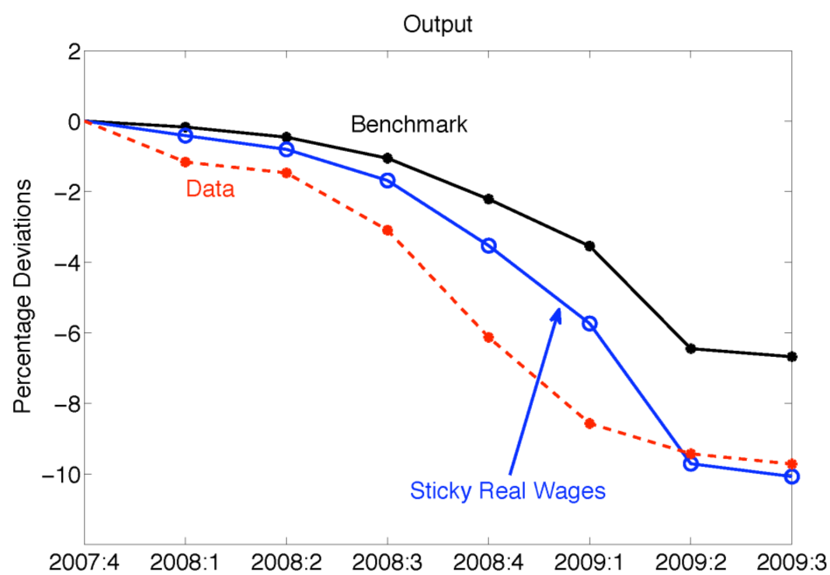


B. Labor

Figure 8: The Great Recession of 2007–2009: Model with Sticky Real Wages

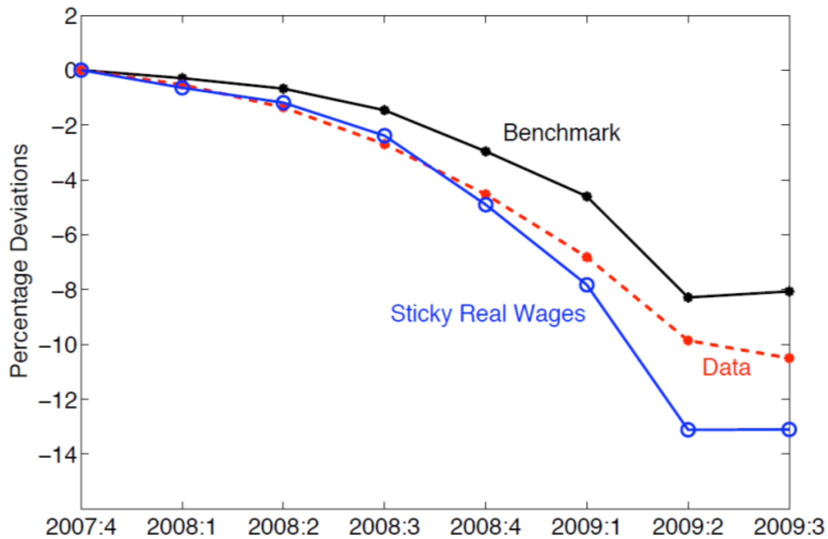


A. Wages



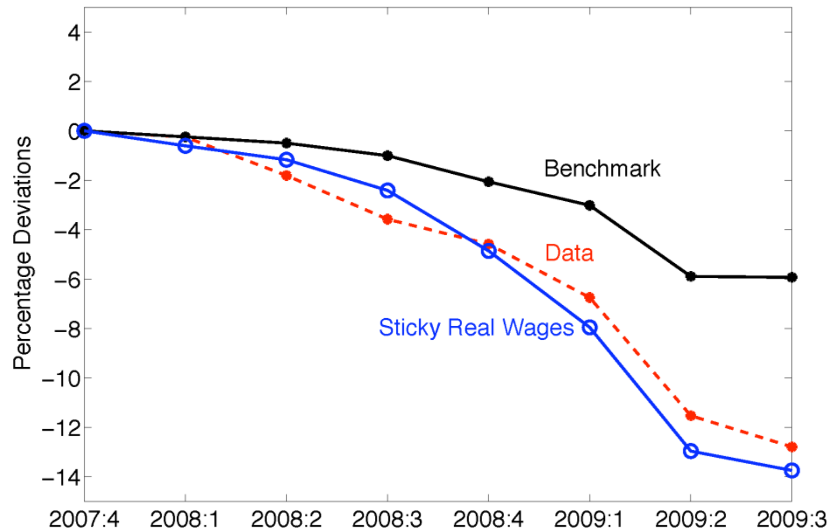
B. Output

Figure 8: The Great Recession of 2007–2009: Model with Sticky Real Wages (Cont.)



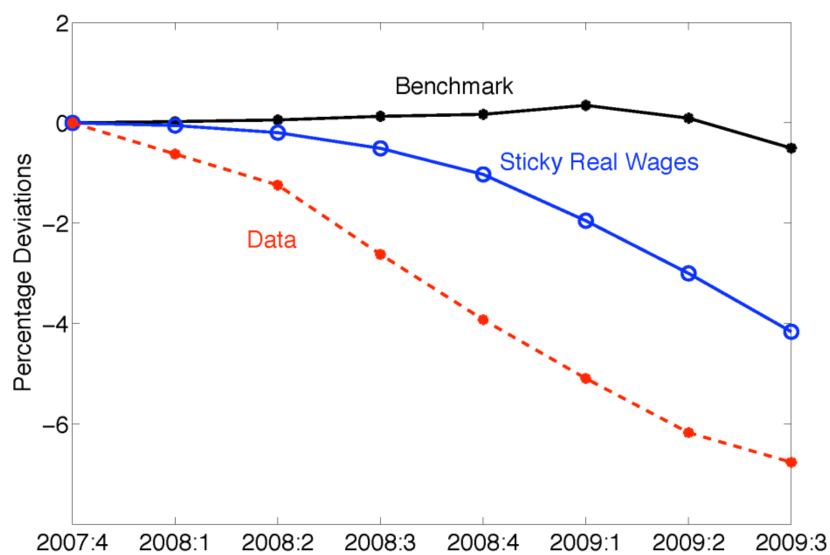
C. Labor

Figure 9: The Great Recession of 2007–2009: Understanding the Labor Wedge



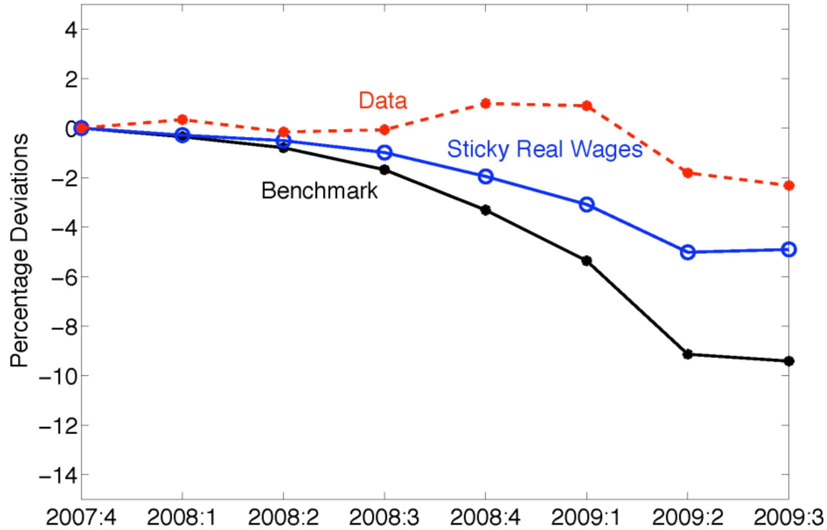
A. Labor Wedge*

*The labor wedge is defined as the ratio of the marginal rate of substitution between leisure and consumption to labor productivity.



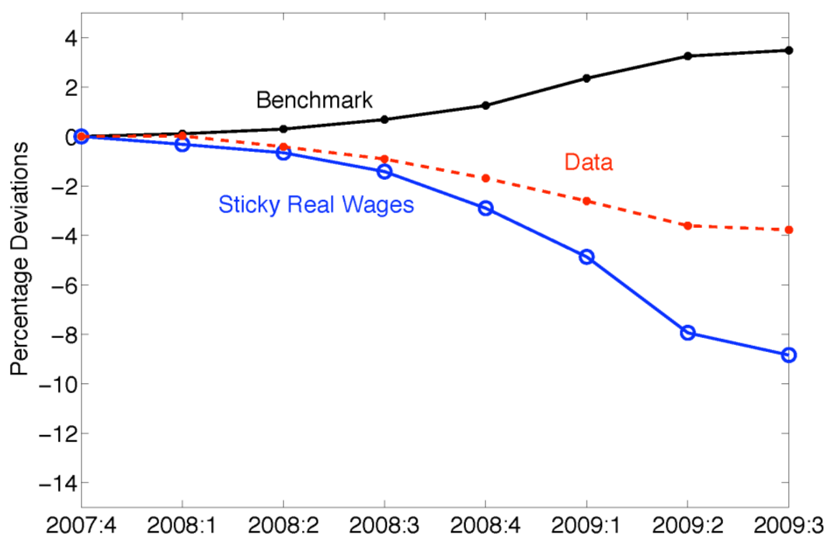
B. Consumption

Figure 9: The Great Recession of 2007–2009: Understanding the Labor Wedge (Cont.)



C. Firm-Side Labor Wedge*

*The firm-side labor wedge is defined as the ratio of the real wage to the marginal product of labor.



D. Consumer-Side Labor Wedge*

*The consumer-side labor wedge is defined as the ratio of the marginal rate of substitution between leisure and consumption to the real wage.