Time-Consistent Public Expenditures

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Preliminary

Motivation

- Question: how should the government choose expenditures over time?
- We propose an answer assuming
  1. proportional taxation of labor and/or capital income;
  2. no commitment technology to set future tax rates; and
  3. period-by-period balanced budgets (no government bonds).
- The model is otherwise canonical: a standard neoclassical setup with utility from both private and public consumption.
- Analysis: a dynamic game between a sequence of governments whose preferences over tax rates disagree.

Our analysis of the dynamic game

- We assume that "reputational mechanisms" are absent and focus on Markov equilibria.

- The model has a state variable. This makes the dynamic game interesting.

- As an additional refinement, we focus on *differentiable* Markov, because there may be many Markov equilibria (Krusell and Smith (2001)). The idea is to characterize the equilibrium for a finite, but long, horizon.

- Our approach is closer to Kydland & Prescott (1977), who sought the limit-of-finite-horizon equilibrium, than to Chari & Kehoe (1990), who followed Abreu, Pearce, & Stacchetti (1990) in using triggers.

Summary of findings I: methods

1. We derive the central first-order necessary condition characterizing the government's tax choice: the *Generalized Euler Equation*, or GEE.

2. The GEE can be interpreted as a dynamic version of public economics principles: the government at t trades off certain distortions at t and $t+1$ against each other.

3. We discuss the sense in which the government at t manipulates it successors.

4. We show how to solve the model.

   (a) We provide some closed-form examples emphasizing (i) the qualitative nature of the results (how a long horizon can be quite different than a short one) and (ii) the connection between finite and infinite horizons.

   (b) Numerical work: even finding a steady state is much harder than in a standard model: it *requires* solving for dynamics. Pinning down a steady state with controlled accuracy is a hard problem.

   (c) We use a natural extension of linearization methods—a "perturbation method"—that happens to be fast and easy to implement.

Summary of findings II: specifics of the public-expenditure problem of which some highlights are

- When the tax base available to the government is capital income—an inelastic source of funds at any moment in time—the government still refrains from taxing at high rates in its efforts to smooth distortions over time.

- We compare the Pareto optimum, Ramsey (= commitment), and Markov allocations.

- The Ramsey and Markov allocations are sometimes, but not always, very different.

- Markov allocations may result in lower tax rates than Ramsey allocations.

The economy

• Standard growth model with a benevolent government, a period-by-period balanced budget, and proportional taxation. In most of the presentation (but not most of the research), the tax base is total income and leisure is not valued.

Households maximize $$\sum_{t=0}^{\infty} \beta^t \, u(c_t, g_t)$$

s.t. $$c_t + k_{t+1} = k_t + (1 - \tau_t) \left[w_t + (r_t - \delta) k_t \right].$$

Resource constraint: $$C_t + K_{t+1} + G_t = f(K_t, 1) + (1 - \delta) K_t$$

Balanced budget constraint: $$G_t = \tau_t \left[f(K_t, 1) - \delta K_t \right].$$

Some useful notation

The following functions are exogenous.

$$\mathcal{G}(K, \tau) \equiv \tau \left[f(K, 1) - \delta K \right]$$

$$\mathcal{C}(K, K', \tau) \equiv f(K, 1) + (1 - \delta)K - K' - \mathcal{G}(K, \tau),$$

where primes denote next-period values.

Choice under commitment: the Ramsey allocation

• Choose $\{\tau_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \, u(\mathcal{C}(K_t, K_{t+1}, \tau_t), \mathcal{G}(K_t, \tau_t))$$

subject to the private sector's first-order condition for savings

$$u_c(\mathcal{C}(K_t, K_{t+1}, \tau_t), \mathcal{G}(K_t, \tau_t)) =$$

$$\beta \, u_c(\mathcal{C}(K_{t+1}, K_{t+2}, \tau_{t+1}), \mathcal{G}(K_{t+1}, \tau_{t+1})) \, [1 + (1 - \tau_{t+1})(f_k(K_{t+1}, 1) - \delta)].$$

for all $t \geq 0$.

• The solution is time-inconsistent in general. Intuitively, the tax rate at t influences savings at earlier dates. When t comes, an opportunity to reset τ_t would lead to a higher value.

Lack of commitment: Markov subgame-perfect equilibria

The government chooses τ and the private sector chooses K'. Key equilibrium objects:

$$\begin{aligned} K' &= \mathcal{H}(K, \tau) \\ \tau &= \Psi(K). \end{aligned}$$

The idea: the government and private agents expect the future governments to obey the *rule* Ψ. In a *one-period deviation* from Ψ, τ is free. Markov requires that no one-period deviation from Ψ yield higher utility for the government.

- What determines these functions?

1. $\mathcal{H}$ must satisfy the FOC for household saving.

2. Ψ must satisfy the government's FOC for taxation.

The functional FOC for household saving

This functional equation is obtained by using $\mathcal{H}$ and Ψ in the household's FOC:

$$u_c\left(\mathcal{C}(K, K', \tau), \mathcal{G}(K, \tau)\right) =$$
$$\beta\, u_c\left(\mathcal{C}\left(K', K'', \tau'\right), \mathcal{G}\left(K', \tau'\right)\right)) \cdot \left\{1 + [1 - \tau'][f_K(K') - \delta]\right\},$$

where K', τ', and K'' are all functions of (K, τ):

$$\begin{aligned} K' &= \mathcal{H}(K, \tau) \\ \tau' &= \Psi(K') = \Psi(\mathcal{H}(K, \tau)) \\ K'' &= \mathcal{H}(K', \tau') = \mathcal{H}(\mathcal{H}(K, \tau), \Psi(\mathcal{H}(K, \tau))) \end{aligned}$$

This functional equation has to hold for all (K, τ): it *defines* $\mathcal{H}$.

Note: Ψ is a determinant of $\mathcal{H}$: the expectations of future government behavior influence how consumers save.

The government's problem and Markov equilibrium

The government's problem:

$$\max_{K',\tau} \; u\left[\mathcal{C}(K,K',\tau),\mathcal{G}(K,\tau)\right] + \; \beta\, v(K') \quad \text{s.t.}$$

$$K' = \mathcal{H}(K,\tau),$$

where

$$v(K) \equiv u\left[\mathcal{C}(K,\mathcal{H}(K,\Psi(K)),\Psi(K)),\mathcal{G}(K,\Psi(K))\right] + \beta\, v[\mathcal{H}(K,\Psi(K))].$$

A Markov equilibrium:

$$\Psi(K) \in \arg\max_{\tau} \left\{u\left[\mathcal{C}(K,\mathcal{H}(K,\tau)),\tau),\mathcal{G}(K,\tau)\right] + \beta\, v(\mathcal{H}(K,\tau))\right\}.$$

To note:

- The government's problem is recursive.

$$v(K) = \max_{\tau} \; u\left[\mathcal{C}(K, \mathcal{H}(K,\tau)), \tau), \mathcal{G}(K,\tau)\right] + \beta \, v[\mathcal{H}(K,\tau)].$$

- Therefore, it has a sequential counterpart:

$$\max_{\{\tau_t, K_{t+1}\}_{t=0}^{\infty}} \; \sum_{t=0}^{\infty} \; \beta^t \, u\left(\mathcal{C}(K_t, K_{t+1}, \tau_t), \mathcal{G}(K_t, \tau_t)\right)$$

subject to $$K_{t+1} = \mathcal{H}(K_t, \tau_t).$$

- Because of the recursive structure, given K', the governments at t and $t+1$ agree on how to set τ_{t+1}.

Deriving the government's FOC for taxes

Given

$$v(K) = \max_{\tau}\ u\left[\mathcal{C}(K, \mathcal{H}(K,\tau)), \tau), \mathcal{G}(K,\tau)\right] + \beta\, v[\mathcal{H}(K,\tau)],$$

find FOC:

$$u_c(\mathcal{C}_{K'}\mathcal{H}_\tau + \mathcal{C}_\tau) + u_g\mathcal{G}_\tau + \beta\left(\frac{\partial v}{\partial K}\right)'\mathcal{H}_\tau = 0.$$

From the DP problem, $\frac{\partial v}{\partial K}$ must equal

$$u_c(\mathcal{C}_K + \mathcal{C}_{K'}(\mathcal{H}_K + \mathcal{H}_\tau\frac{\partial\Psi}{\partial K}) + \mathcal{C}_\tau\frac{\partial\Psi}{\partial K}) + u_g(\mathcal{G}_K + \mathcal{G}_\tau\frac{\partial\Psi}{\partial K}) + \beta(\frac{\partial v}{\partial K})(\mathcal{H}_K + \mathcal{H}_\tau\frac{\partial\Psi}{\partial K})$$

The envelope theorem applies, delivering

$$\frac{\partial v}{\partial K} = u_c(\mathcal{C}_K + \mathcal{C}_{K'}\mathcal{H}_K) + u_g\mathcal{G}_K + \beta\left(\frac{\partial v}{\partial K}\right)'\mathcal{H}_K = 0.$$

Insert the expression for $(\frac{\partial v}{\partial K})'$ from the FOC, to deliver

$$\frac{\partial v}{\partial K} = u_c(\mathcal{C}_K + \mathcal{C}_{K'}\mathcal{H}_K) + u_g\mathcal{G}_K - \frac{\mathcal{H}_K}{\mathcal{H}_\tau}\left(u_c(\mathcal{C}_{K'}\mathcal{H}_\tau + \mathcal{C}_\tau) + u_g\mathcal{G}_\tau\right).$$

Finally, update to $(\frac{\partial v}{\partial K})'$ and substitute back into the FOC:

$$u_c(\mathcal{C}_{K'}\mathcal{H}_\tau + \mathcal{C}_\tau) + u_g\mathcal{G}_\tau +$$

$$\beta\mathcal{H}_\tau\left\{u_c'(\mathcal{C}_K' + \mathcal{C}_{K'}'\mathcal{H}_K') + u_g'\mathcal{G}_K' - \frac{\mathcal{H}_K'}{\mathcal{H}_\tau'}\left(u_c'(\mathcal{C}_{K'}'\mathcal{H}_\tau' + \mathcal{C}_\tau') + u_g'\mathcal{G}_\tau'\right)\right\} = 0.$$

This is the GEE, for what it's worth.

Our time-consistent equilibrium

The GEE has to hold for all K, and is a *functional equation* determining $\Psi(K)$, given $\mathcal{H}(K,\tau)$.

Equilibrium: A *time-consistent policy equilibrium* is a set of differentiable functions Ψ and $\mathcal{H}$ such that

- $\mathcal{H}(k,\tau)$ solves the functional FOC of the private sector; and
- $\Psi(K)$ solves the functional FOC of the government.

Interpretations I

• The **"raw"** version of the FOC (same as above). Using the definition $\mathcal{C}(K, K', \tau) = f(K, 1) + (1 - \delta)K - K' - \mathcal{G}(K, \tau)$, we obtain

$$0 = u_c \, [-\mathcal{H}_\tau - \mathcal{G}_\tau] + u_g \, \mathcal{G}_\tau +$$

$$\beta \, \mathcal{H}_\tau \left\{ u'_c \, [f'_K + 1 - \delta - \mathcal{H}'_K - \mathcal{G}'_K] + u'_g \, \mathcal{G}'_K - \frac{\mathcal{H}'_K}{\mathcal{H}'_\tau} \left(u'_c [-\mathcal{H}'_\tau - \mathcal{G}'_\tau] + u'_g \, \mathcal{G}'_\tau \right) \right\}.$$

The term $\mathcal{H}'_K / \mathcal{H}'_\tau$ reflects the **variational** nature of the FOC: how to vary optimally τ and τ' subject to keeping K and K'' unchanged. Thus, $-\mathcal{H}'_K / \mathcal{H}'_\tau$ is the increase in τ' needed in order not to change K''.

Interpretations II

The **"public economics"** version of the FOC. Trade off wedges:

$$\mathcal{G}_\tau \left[u_g - u_c \right] + \mathcal{H}_\tau \left[-u_c + \beta u'_c (1 + f'_K - \delta) \right] + \beta \mathcal{H}_\tau \left(\mathcal{G}'_K - \frac{\mathcal{H}'_K}{\mathcal{H}'_\tau} \mathcal{G}'_\tau \right) \left[u'_g - u'_c \right] = 0.$$

Leisure

With valued leisure, we need another function: $\mathcal{L}(K, \tau)$. It will be defined by the private sector's FOC for the labor-leisure choice.

The FOC for taxes, in its public economics version, will now contain the additional terms

$$\mathcal{L}_\tau \left[u_c f_L - u_\ell \right] + \beta \mathcal{H}_\tau \cdot \left\{ \mathcal{L}'_K - \frac{\mathcal{H}'_K}{\mathcal{H}'_\tau} \mathcal{L}'_\tau \right\} \left[u'_c f'_L - u'_\ell \right]$$

An alternative equilibrium definition

No distinction between government and private sector.

Equilibrium objects: $h(K)$ and $\psi(K)$, defined by the solution to

$$v(K) = \max_{K',\tau} \{u\,[\mathcal{C}(K,K',\tau),\mathcal{G}(K,\tau)] + \beta\, v(K')\}$$

subject to

$$u_c\,(\mathcal{C}(K,K',\tau),\mathcal{G}(K,\tau)) =$$

$$\beta\, u_c\,(\mathcal{C}\,(K',\mathcal{H}(K'),\Psi(K')),\mathcal{G}\,(K',\Psi(K')))\cdot\{1+[1-\Psi(K')][f_K(K')-\delta]\}\,.$$

- With $h(K) \equiv \mathcal{H}(K,\Psi(K))$ and $\psi(K) = \Psi(K)$ for all K, it is easy to see that the two equilibrium definitions are equivalent.

- This definition is useful for computation.

A closed-form example

Suppose that utility is log-log-log, production is Cobb-Douglas, and depreciation is 100%:

$$u(c, \ell, g) = \alpha \ln c + (1 - \alpha) \ln(1 - \ell) + \gamma \ln g$$

and

$$f(K, L) = A \cdot K^{\theta} L^{1-\theta}.$$

- With $\alpha = 1$ (no leisure) and capital income taxation, we obtain

$$\Psi(K) = \frac{\gamma(1-\beta\theta)}{\theta(1+\gamma)}$$

and

$$\mathcal{H}(K,\tau) = \beta\theta\frac{\theta(1+\gamma)-\gamma+\gamma\beta\theta}{1+\gamma\beta\theta}(1-\theta\tau)K^{\theta}.$$

  - For $\dfrac{\gamma}{\theta(1+\gamma)} \geq 1$, however, this is not the limit of finite-horizon equilibria; rather, savings are zero and capital taxes are over 100%.
  - For $\dfrac{\gamma}{\theta(1+\gamma)} < 1$, this is the limit of finite-horizon equilibria.
  - This equilibrium does not coincide with the Ramsey allocation.

- With $\alpha \in (0,1)$ (valued leisure) and a general income tax,

$$\Psi(K) = \frac{\gamma(1-\beta\theta)}{\alpha+\gamma},$$

$$\mathcal{H}(K,\tau) = \beta\theta(1-\tau)K^{\theta}\left(\frac{\alpha}{\alpha+(1-\alpha)(1-\beta\theta)}\right)^{1-\theta},$$

and

$$\mathcal{L}(K,\tau) = \frac{\alpha}{\alpha+(1-\alpha)(1-\beta\theta)}.$$

  - Similarly, this is not always the limit of finite-horizon equilibria.
  - This equilibrium coincides with the Ramsey allocation.

Computation of a steady state

•• We need to find functions that jointly satisfy the two functional FOCs.

•• Can we find a steady state by just evaluating at $\bar{K}$ and $\bar{\tau}$? No: 2 equations and 4 unknowns: $\mathcal{H}_k$ and $\mathcal{H}_\tau$ appear.

•• Can we find these derivatives by differentiation of the FOC for savings?

- No, because then the derivative of Ψ appears: we cannot solve for a steady state without solving for dynamics.
- To obtain a derivative of Ψ, one could differentiate the FOC for taxes.
- But then the second derivatives of $\mathcal{H}$ appears: to solve for first-order dynamics, one needs second-order dynamics, and so on.

•• Numerical solution: use a "generalization of linearization methods"—a Perturbation method.

Idea: Focus on finding h and ψ by using an nth-order polynomial. Successively differentiate the functional equations.

$$0 = EE^{(0)}\left(h^{(0)}, \psi^{(0)}\right) \tag{2}$$

$$0 = GEE^{(0)}\left(h^{(0)}, \psi^{(0)}, h^{(1)}, \psi^{(1)}\right) \tag{3}$$

$$0 = EE^{(1)}\left(h^{(0)}, \psi^{(0)}, h^{(1)}, \psi^{(1)}\right) \tag{4}$$

$$0 = GEE^{(1)}\left(h^{(0)}, \psi^{(0)}, h^{(1)}, \psi^{(1)}, h^{(2)}, \psi^{(2)},\right) \tag{5}$$

$$\ldots$$

$$0 = EE^{(n)}\left(h^{(0)}, \psi^{(0)}, h^{(1)}, \psi^{(1)}, \cdots, h^{(n)}, \psi^{(n)}\right) \tag{6}$$

$$0 = GEE^{(n)}\left(h^{(0)}, \psi^{(0)}, h^{(1)}, \psi^{(1)}, \cdots, h^{(n)}, \psi^{(n)}, 0, 0\right) \tag{7}$$

Guess on $h^{(0)}$. Solve for $\psi^{(0)}$. Then successively solve for $(h^{(i)}, \psi^{(i)})$ Pers, $i = 1, \ldots, n$, and finally check that the last equation holds.

Quantitative analysis: baseline example

We specify the period utility function as

$$u(c, \ell, g) = (1 - \alpha_p)\alpha_c \ln c + (1 - \alpha_p)(1 - \alpha_c) \ln \ell + \alpha_p \ln g$$

The production function is

$$f(K, L) = A \cdot K^{\theta} L^{1-\theta}.$$

Parameter Values		
$\theta = .36$	$\alpha_c = .30$	$\alpha_p = .13$
$\beta = .96$	$\delta = .08$	

Table 1: Parameterization of the Baseline Model Economy.

Capital taxes only

Statistic	Pareto	Ramsey	Markov
Y	1.000	0.588	0.488
K/Y	2.959	1.734	1.193
C/G	2.005	4.779	3.211
L	0.350	0.278	0.255
τ	–	0.673	0.812

Very large taxes. Small expenditures in Ramsey. Also small in Markov, even though τ is lump-sum: a decrease in τ increases K'.

Labor taxes only

Statistic	Pareto	Ramsey	Markov
Y	1.000	0.700	0.719
K/Y	2.959	2.959	2.959
C/G	2.005	2.005	3.017
L	0.350	0.245	0.252
τ	–	0.397	0.297

No intertemporal distortion. Ramsey has the right ratio between C and G. Markov does not: it ignores the positive effect of a higher τ_t on L_{t-1}.

Total income taxes

Statistic	Pareto	Ramsey	Markov
Y	1.000	0.669	0.693
K/Y	2.959	2.527	2.649
C/G	2.005	2.005	2.928
L	0.350	0.256	0.258
τ	–	0.334	0.255

Markov again taxes less; it does not take into account the effect of the tax on yesterday work effort, and it uses τ strategically.

Conclusions

•1 We derived a FOC for taxes to help interpret the decision of governments with (partly) conflicting objectives: an equation that allows qualitative and quantitative interpretations.

•2 We show how to solve this functional equation; a much harder problem than that of solving a standard Euler equation. The numerical methods seem to work very well.

•3 We document some interesting properties for the problem of optimal provision of public goods: Markov does not necessarily tax more heavily than Ramsey, and the difference between the two is nontrivial.

•4 The class of problems for which these methods are relevant seems large.

•5 Some remaining issues: bonds, other assumptions on the set of tax instruments, existence.