INCOME AND WEALTH HETEROGENEITY IN THE MACROECONOMY:

Aggregate Shocks and Distributional Issues

Material from papers by Per Krusell and Tony Smith

Motivation:

1. Macroeconomics founded on representative agent: poor "microfoundation".

2. Traditional (say, Keynesian) models reflect a concern about inequality, not just efficiency. To properly address such a concern, a heterogeneous-agent framework is needed.

To explore:

a set of macro models with consumer heterogeneity due to lack of complete insurance against labor market outcomes.

Question asked here:

How different is the heterogeneous-agent model (for business cycles and asset prices)?

Related questions that come up:

- What are the origins of the observed inequality in income and wealth?
- What are the welfare consequences of business cycles for different groups in society? How do various kinds of economic policy affect the welfare of different groups?

To go over:

- Modeling: standard macro/business cycle setup, except
  - There is a large number of consumers, each facing idiosyncratic income risk.
  - Insurance markets are replaced with a small set of assets allowing some self-insurance (capital and a bond).

  The benchmark model is therefore essentially Aiyagari (*QJE* 1994) plus aggregate risk.

- Model solution:
  - One can do it!
  - Aggregate variables depend (almost) only on the mean: "approximate aggregation".
  - Check a large number of variations of the model—it always works.
  - Explanation for the approximate aggregation.

- Problem: the model's wealth distribution is counterfactual. There are too few poor, and the fat cats are not fat enough. This issue is nontrivial.

- To fix the problem:
  - use preference heterogeneity (in thrift, without too much cheating),
  - the wealth distribution now looks OK,
  - and we still get approximate aggregation.
  - Now, the model's aggregate time series properties are quite different: it's not a permanent-income model any more.

- Auxiliary stuff: asset pricing implications (extension to Heaton/Lucas and others)
  - Strict borrowing constraints for bonds $\Rightarrow$ empirically reasonable (high) market price of risk.

Outline:

1. benchmark model
2. basic result
3. robustness
4. explanation for approximate aggregation
5. problem with wealth distribution
6. how we amend the model
7. new wealth distribution
8. business cycle statistics
9. asset prices

THE MODEL

Population

There is a large number (measure 1) of ex-ante identical agents.

Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

with

$$u(c) = \lim_{\nu \to \sigma} \frac{c^{1-\nu} - 1}{1 - \nu}.$$

Uncertainty

- Aggregate shock, z: $z = z_g$ (good times), or $z = z_b$ (bad times), with first-order Markov structure given by

$$\begin{pmatrix} \pi_{gg} & \pi_{bg} \\ \pi_{gb} & \pi_{bb} \end{pmatrix}.$$

- Individual employment shock, ϵ: $\epsilon = 1$ (employed), or $\epsilon = 0$ (unemployed).

- Amount of labor supplied when employed: $\tilde{l}$

- The individual and aggregate shocks are correlated: the number of unemployed always equals u_g in good times and u_b in bad times; controlling for z, individual shocks are uncorrelated.

- Markov structure on (z, ϵ):

$$\Pi' = \begin{pmatrix} \pi_{g1|g1} & \pi_{g1|b1} & \pi_{g1|g0} & \pi_{g1|b0} \\ \pi_{b1|g1} & \pi_{b1|b1} & \pi_{b1|g0} & \pi_{b1|b0} \\ \pi_{g0|g1} & \pi_{g0|b1} & \pi_{g0|g0} & \pi_{g0|b0} \\ \pi_{b0|g1} & \pi_{b0|b1} & \pi_{b0|g0} & \pi_{b0|b0} \end{pmatrix}.$$

Technology

- Aggregate output is produced and used according to

$$\bar{c} + \bar{i} = \bar{y} = z\,\bar{k}^{\alpha}\bar{l}^{1-\alpha},$$

with $\bar{m}$ denoting the population average of m.

- Capital accumulates according to

$$\bar{k}' = (1-\delta)\bar{k} + \bar{i}.$$

Markets

- There are no insurance markets.
- The individual can use capital or bonds for self-insurance: $(k, b) \in \mathcal{K} \times \mathcal{B} \equiv [\underline{k}, \infty) \times [\underline{b}, \infty)$.
- Market clearing: capital holdings sum up to total capital, and bonds sum to zero.
- The rental rate is $r(\bar{k}, \bar{l}, z)$ and the wage rate is $w(\bar{k}, \bar{l}, z)$ (first-order conditions to firm's static problem); the price of bonds is q.

Cases when the aggregation theorem holds

This theorem holds if

- all agents have the same utility function;
- this utility function is time-separable and of the generalized Bergson form (leisure OK to include, if of the form we look at below);
- markets are complete.

Recursive Competitive Equilibrium

- Let Γ denote the current measure of consumers over holdings of wealth and employment status.
- The relevant aggregate state is (Γ, z).
- The individual state is $(\omega, \epsilon; \Gamma, z)$.
- Let H denote the equilibrium transition function for Γ:

$$\Gamma' = H(\Gamma, z, z').$$

Consumers

$$v(\omega, \epsilon; \Gamma, z) = \max_{c, k', b'} \{u(c) + \beta E[v(\omega', \epsilon'; \Gamma', z') | z, \epsilon]\}$$

subject to:

$$c + k' + b' = \omega$$

$$\omega' = r(\bar{k}', \bar{l}', z')k' + q(\Gamma', z')b' + w(\bar{k}', \bar{l}', z')\, \tilde{l}\, \epsilon' + (1 - \delta)k'$$

$$\Gamma' = H(\Gamma, z, z')$$

$$(k', b') \geq (\underline{k}, \underline{b})$$

and a law of motion for (z, ϵ).

$\Rightarrow$ optimal decision rules:

$$k' = f(\omega, \epsilon; \Gamma, z).$$

$$b' = g(\omega, \epsilon; \Gamma, z).$$

A **recursive competitive equilibrium** is a law of motion H, individual functions (v, f, g), and pricing functions (r, w, q) such that:

1. (v, f, g) solves the consumer's problem.
2. (r, w) is competitive.
3. H is generated by f and g and the law of motion for (z, ϵ).
4. Bond markets clear (the sum of the g's over the population equals zero).

Stationary Stochastic Equilibrium

A stationary equilibrium is a recursive competitive equilibrium described by an ergodic set $\mathcal{D}$ of distributions and an invariant probability measure $\mathcal{P}$ over this set.

COMPUTATION: EXPLOITING BOUNDED RATIONALITY

- We assume that agents have incorrect perceptions regarding how the economy works: agents only think prices depend on a finite set of moments of Γ: $m \equiv (m_1, m_2, \ldots, m_I)$.
- The function H is then represented by the function H_I: $m' = H_I(m, z, z')$.
- Given that agents behave based on these perceptions, derive the implied aggregate behavior and check the extent to which the agents' perceptions differ from how the economy behaves.

We describe the iterative procedure used to approximate stationary equilibria as follows (an example without bonds):

1. Select I.
2. Guess on H_I in the form of some given parameterized functional form, and guess on parameter values.
3. Solve the consumer's problem given H_I, and obtain the implied decision rules f_I.
4. Use these decision rules to simulate the behavior of N agents (with N a large number).
5. Use the "stationary region" of the simulated data to estimate a new set of parameters for H_I.
6. Update the parameters/iterate until a fixed point in these parameters is found. At this stage, we obtain a goodness-of-fit.
7. If the goodness-of-fit is satisfactory, stop. If it is not satisfactory, increase I, or, as a less ambitious step, try a different functional form for H_I.

Example (no bonds again)

$I = 1$ and H_I is linear:

$$\begin{aligned} z = z_g : \ \bar{k}' &= a_0 + a_1 \bar{k} \\ z = z_b : \ \bar{k}' &= b_0 + b_1 \bar{k} \end{aligned}$$

The agent solves the following problem:

$$v(k, \epsilon; \bar{k}, z) = \max_{c,\, k'} \{ u(c) + \beta E[v(k', \epsilon'; \bar{k}', z') | z, \epsilon] \}$$

subject to:

$$\begin{aligned} & c + k' = r(\bar{k}, \bar{l}, z)k + w(\bar{k}, \bar{l}, z)\, \tilde{l}\, \epsilon + (1 - \delta)k \\ & \bar{k}' = a_0 + a_1 \bar{k} \text{ if } z = z_g \\ & \bar{k}' = b_0 + b_1 \bar{k} \text{ if } z = z_b \\ & \text{Markov law of motion for } (z, \epsilon) \\ & k' \geq 0 \end{aligned}$$

$\Rightarrow$ optimal decision rule:

$$k' = f(k, \epsilon; \bar{k}, z).$$

Chief computational task: Find the fixed point $(a_0^\star, a_1^\star, b_0^\star, b_1^\star)$.

Solving the consumer's problem

General idea: Approximate each of the functions $v(k, 1; \bar{k}, z_g)$, $v(k, 1; \bar{k}, z_b)$, $v(k, 0; \bar{k}, z_g)$, and $v(k, 0; \bar{k}, z_b)$ on a grid of points in the $(k,\bar{k})$ plane. Use cubic spline and polynomial interpolation to compute the value function at points not on the grid. Choices for capital are not restricted to the grid. The approach is similar to value iteration on a discrete grid (see Johnson et al. (1993)).

1. Choose a grid. Choose an initial set of values for each of the above functions above at each of the grid points.
2. Compute cubic splines using the initial set of values.
3. For each of the four (z, ϵ) pairs, maximize the right-hand side of the Bellman equation at each point in the grid. Record the new optimal value at this grid point. (Use Newton-Raphson optimization routine at grid points for which the borrowing constraint does not bind.)
4. Replace the initial values with the new optimal values. Repeat steps 2 and 3 until the new and old values are close.

FINDINGS

PARAMETER VALUES

A. Model Parameters

- Quarterly model: $\beta = 0.99$, $\delta = 0.025$.
- σ (CRRA) = 1,2,3,4,5
- α (capital share) = 0.36
- average duration of good times = 8 periods
- average duration of bad times = 8 periods
- value of technology shock in good times = 1.01
- value of technology shock in bad times = 0.99
- unemployment rate in good times = 0.04
- unemployment rate in bad times = 0.10
- average duration of an unemployment spell in good times = 1.5
- average duration of an unemployment spell in bad times = 2.5

So:

$$\Pi' = \begin{pmatrix} 0.851 & 0.123 & 0.583 & 0.094 \\ 0.116 & 0.836 & 0.031 & 0.350 \\ 0.024 & 0.002 & 0.292 & 0.031 \\ 0.009 & 0.039 & 0.094 & 0.525 \end{pmatrix}.$$

B. Solution algorithm/simulation parameters

- Solving the agent's problem: We use one of many available methods (use cubic splines to build an approximation to the value function).
- Simulations: We simulate the behavior of 5,000 agents over 11,000 periods (first 1,000 periods are dropped). Initial condition: all agents have the same wealth (results are not sensitive to this initial condition).

MODEL BEHAVIOR

Good times:

$$\log \bar{k}' = 0.095 + 0.962 \log \bar{k}$$

$$R^2 = 0.999998 \quad \hat{\sigma} = 0.0028\%$$

Bad times:

$$\log \bar{k}' = 0.085 + 0.965 \log \bar{k}$$

$$R^2 = 0.999998 \quad \hat{\sigma} = 0.0036\%$$

- Very good fit with only one moment. Evidence:
  1. The law of motion for aggregate capital track the data extremely well.
  2. The agents' ability to forecast prices is extremely high.
  3. Would more moments help in these forecasts? No.

- How do the results change if we let agents perceive that prices depend on an additional moment, i.e. use $I = 2$? They (almost) do not change.

Forecasting accuracy				
	1 quarter ahead		*25 years ahead*	
Variable	*corr*$(x, \hat{x})$	*max % error*	*corr*$(x, \hat{x})$	*max % error*
Capital	0.999999	0.0143	0.999614	0.2373
Rental rate	1.000000	0.0091	0.999888	0.152
Wage rate	0.999999	0.0051	0.999583	0.0855

Include standard deviation, skewness, and kurtosis

Good times:

$$\log \bar{k}' = 0.092 + 0.963 \log \bar{k} + \underset{(69.9)}{0.00087 s_2} - \underset{(-18.0)}{0.00018 s_3} + \underset{(11.9)}{0.00011 s_4}$$

$$R^2 = 0.999999 \quad \text{(before: 0.999998)}$$
$$\hat{\sigma} = 0.0018\% \quad \text{(before: 0.0028\%)}$$

Bad times:

$$\log \bar{k}' = 0.081 + 0.965 \log \bar{k} + \underset{(57.5)}{0.0012 s_2} - \underset{(-23.3)}{0.00029 s_3} + \underset{(15.3)}{0.00019 s_4}$$

$$R^2 = 0.999999 \quad \text{(before: 0.999998)}$$
$$\hat{\sigma} = 0.0024\% \quad \text{(before: 0.0036\%)}$$

Model solved with two moments

Good times:

$$\log \bar{k}' = 0.094 + 0.963 \log \bar{k} + 0.00016 \log s_2$$

$$R^2 = 0.999999 \quad \hat{\sigma} = 0.0019\%,$$

$$\log s_2' = 0.048 - 0.019 \log \bar{k} + 0.999 \log s_2$$

$$R^2 = 0.9998 \quad \hat{\sigma} = 0.043\%$$

Bad times:

$$\log \bar{k}' = 0.084 + 0.965 \log \bar{k} + 0.00030 \log s_2$$

$$R^2 = 0.999999 \quad \hat{\sigma} = 0.0026\%,$$

$$\log s_2' = 0.057 - 0.019 \log \bar{k} + 0.994 \log s_2$$

$$R^2 = 0.9995 \quad \hat{\sigma} = 0.073\%$$

Why doesn't the distribution matter?

- Are the marginal propensities to consume and work the same across all individuals?
  1. The decision rules for capital are close to linear $\Rightarrow$ most agents with the same employment status have the same savings propensities.
  2. Across agents with different employment status, the marginal propensities to save differ, but by very little.

- Although the propensities to consume do not differ by much, they do differ: it is possible to redistribute capital to get heterogeneity to matter much more. Moreover, the distribution of capital does move significantly over time.

  However: propensities are significantly different only for the very poorest people, and their effect on $\bar{k}'$ is (almost) nil.

- Why are the marginal propensities so similar? This has to do with the utility costs of variations in consumption:
  - Lucas (1987), Cochrane (1989), and others: in representative-agent models, they are very small.
  - Asset pricing literature (Heaton and Lucas (1996) & Co.): with idiosyncratic risks, one asset does very well in terms of providing insurance (in utility terms; here, consumption is much more variable for individuals than for aggregate).

VARIATIONS

We explored alternative setups:

1. Changing parameters in the baseline model:
   (a) Parameters which can be changed without changing the main finding: the degree of risk aversion, the borrowing constraints, the duration of unemployment.
   (b) Lower discount factors (more impatience) weaken the result (consistent with Bewley (1977)).

2. Other types of heterogeneity added:
   (a) two agents with different discount factors;
   (b) two agents with different degrees of risk aversion.

   Once again, only the mean matters.

3. Valued leisure (the RBC): Same as baseline model.

4. Heterogeneity in income processes (Castañeda, Díaz-Giménez, and Ríos-Rull (1995)): Same as baseline model.

5. Fixed costs of adjusting capital (forcing differences in propensities to save): Same as baseline model.

THE WEALTH DISTRIBUTION: A PROBLEM

The distribution of wealth							
	% of wealth held by top					Fraction with	Gini
	1%	5%	10%	20%	30%	wealth < 0	coefficient
$\underline{b} = 0$ model	3%	11%	20%	35%	47%	0%	0.26
$\underline{b} = -2.4$ model	3%	13%	23%	39%	52%	0.5%	0.33
Data	30%	51%	64%	79%	88%	11%	0.79

We need (i) the rich to save more or (ii) have higher returns to their savings.

We focus on the first of these:

- Difference in discount factors (on average, the rich families are the patient ones).
- Unemployment insurance (9% of average wage; creates disincentive for poor to save).

Preference Heterogeneity Parameters

Idea: imperfect passing on of genes across generations.

- Three values of $\tilde{\beta}$: 0.9858, 0.9894, and 0.9930.
- Invariant distribution: 10% at each of the extreme values of $\tilde{\beta}$, 80% at middle value.
- No immediate transitions between extreme values.
- Average duration of highest and lowest $\tilde{\beta}$'s is 50 years (roughly matching the length of a generation).

The distribution of wealth							
	% of wealth held by top					Fraction with	Gini
	1%	5%	10%	20%	30%	wealth < 0	coefficient
$\underline{b} = 0$ model	20%	46%	61%	74%	79%	0%	0.66
$\underline{b} = -2.4$ model	23%	55%	73%	87%	92%	11%	0.82
Data	30%	51%	64%	79%	88%	11%	0.79

APPROXIMATE AGGREGATION STILL WORKS. There are many poor, but they have no wealth. Those with wealth are well insured in utility terms.

Why are the poor poor in this model?

CHALLENGING PERMANENT INCOME THEORY

The aggregate time series				
Model	mean(k_t)	corr(c_t, y_t)	std.dev.(i_t)	corr(y_t, y_{t-4})
Complete markets	11.54	0.691	0.031	0.486
Benchmark	11.61	0.701	0.030	0.481
$\sigma = 5$	12.32	0.741	0.033	0.524
RBC	11.58	0.669	0.027	0.339
Stochastic β	11.78	0.825	0.027	0.459

THE MARKET PRICE OF RISK

Use the Euler equations of an agent who holds both assets to get:

$$E_{z',\epsilon'|z,\epsilon,I}[m'(R'_e - R_f)] = 0.$$

Use the law of iterated expectations to get:

$$E_{z'|z,\epsilon,I}[(R'_e - R_f)(E_{\epsilon'|z',z,\epsilon,I}[m'])].$$

Following Hansen and Jagannathan (1991):

$$\frac{E_{z'|z}(R'_e - R_f)}{\sigma_{z'|z}(R'_e - R_f)} = A \times B$$

where

$$\begin{aligned} A &= -\mathrm{cor}_{z'|z}(R'_e - R_f, E_{\epsilon'|z,\epsilon,z'}[m]') \\ B &= \frac{\sigma_{z'|z}(E_{\epsilon'|z,\epsilon,z'}[m'])}{E_{z'|z}(E_{\epsilon'|z,\epsilon,z'}[m'])} \end{aligned}$$

B is an upper bound on the market price of risk. In our model, $A = -1$, so the bound is exact. Using S&P 500 returns, Lettau and Uhlig (1996) report a market price of risk of 0.27.

Results for Market Price of Risk

- Complete markets: 0.0022
- No preference shocks, $\underline{b} = -2.4$: 0.0025
- Preference shocks, $\underline{b} = -2.4$: 0.0026
- No preference shocks, $\underline{b} = 0$: 0.211
- Preference shocks, $\underline{b} = 0$: 0.214

WELFARE COSTS
THE ISSUE

People seem puzzled by the low estimates of the welfare costs of business cycles that come from neoclassical models (main reference Lucas (1987)). Should we be puzzled?

Unless the public debate is completely misguided, we probably should.

What's wrong with Lucas's calculations, then?

REVIEW: THE LUCAS CALCULATION

Lucas used a very simple model:

1. He assumed that eliminating cycles amounts to removing all but the *trend part* of the per-capita consumption stream observed in postwar U.S. data; and

2. to get a welfare measure, he assumed a representative agent with time-separable, expected-utility preferences, calibrated as in the applied dynamic macro literature.

LUCAS'S RESULT: the gains from eliminating fluctuations are extremely small: in terms of a permanent change in the flow of consumption, the gain is around 0.008% of average consumption (log utility).

Comments on Lucas's approach:

1. Lucas's approach is neoclassical in the sense that cycles are "neutralized": both recessions and booms are eliminated.

2. *How* business cycles could be avoided is not spelled out. The welfare cost estimate is thus subject to the "Lucas critique". However, it may constitute an upper bound on the costs.

So, what could be wrong with this calculation?

There seem to be three possible routes to take:

- Dispute the assumption that no output can be gained on average (take a Keynesian view).

- Make alternative assumptions about the time series representation of postwar data (maybe consumption is not trend-stationary) or about the preferences of the representative agent (maybe time-separability is a bad assumption, or the expected utility assumption is inappropriate).

- Reinterpret the public concern about cycles: perhaps the costs of cycles really are small for the representative—in the sense of average—consumer, but large or very large for some consumers (the unlucky/poor/unemployed).

The focus here: the last of these three routes.

TASK:

Use the above model with consumer heterogeneity to estimate the welfare costs of business cycles for different consumers. Key properties of the model:

- Consumers differ in tastes, wealth, and employment (but not in productivity/wage).

- There is individual-specific risk, and there are incomplete markets: aggregate capital is the only asset.

- The calibrated model has *substantial* wealth heterogeneity, and *substantial* movements in individual consumption. This is potentially important: poor consumers are *de facto* less well-insured, and therefore they might suffer much more from risk.

- Agents can be "long-term" unemployed; the group of long-term unemployed is particularly large in recessions.

RESULTS, IN BRIEF:

- We argue that, if one neutralizes cycles in Lucas's spirit, it is hard to claim that employment risk decreases if cycles are eliminated.

- It follows that the risk that remains is price risk (wages and rates of return). Some general points regarding price risk:

  - It gives very small effects on welfare for almost all agents.

  - Its qualitative effects are somewhat surprising: depending on the individual's situation, price risk may be liked or disliked.

- The very poorest of the poor can gain up to several percentage points from eliminating cycles (in terms of long-run consumption equivalents). *However*: there are vanishingly few agents who are this poor (say, a handful in an economy the size of that of the U.S.), and just a tiny bit of wealth wipes out the gain.

- Agents with more wealth do not necessarily like cycles more; the relation is sometimes nonmonotonic.

- On average, the gains from eliminating cycles are very low, and sometimes negative.

- In sum, we do not resolve the puzzle; if anything, we make it worse.

A few comments on the literature:

No paper has looked at the differential effects of cycles across the population, our main focus. No paper has considered "realistic" wealth distributions/individual consumption fluctuations.

Points made so far:

- The representative-agent assumption was relaxed in İmrohoroğlu (1989). Idea: with incomplete insurance against individual-specific employment shocks, individual consumption fluctuates much more than per-capita consumption and, on the margin, aggregate fluctuations hurt more. Exogenous, constant prices. RESULT: increases average costs, but by a small amount.

- Other preferences/time series representations: Obstfeldt (1994) and later others (Tallarini (1997), Dolmas (1998)) considers time-nonseparable (à la Epstein-Zin) "risk-sensitive" preferences. RESULT: increases costs, but not by that much (unless discount factor is very close to 1 and per-capita consumption growth rate follows a random walk).

- Atkeson and Phelan (1994): argue, as we do, that eliminating cycles may not reduce the risk in the idiosyncratic processes. They also study the connection to the equity premium puzzle: to the extent stock market fluctuations are business cycle fluctuations, the market price of risk on stock gives the marginal cost to consumers of cycles. This number is big. RESULT: unless bond prices fluctuate a lot, hard to get large costs.

- Beaudry and Pages (1996) focus on wage shocks and argue that the wage after unemployment is significantly lower than before unemployment. Further, they assume that business cycles are the cause of all frictions and that there would be no idiosyncratic wage movements if business cycles were eliminated. RESULT: larger numbers.

- Gomes, Greenwood, and Rebelo (1998) study a search-unemployment environment. RESULT: the gains from eliminating cycles are negative, because risk to a searcher is good.

What does it mean to eliminate cycles?

- In general, who knows? We don't have a THEORY of fluctuations, nor a theory of how to eliminate them.

- Aggregate variables: Lucas-style approach
  - We replace z by its mean.
  - We replace u by its mean.
  - This does not result in output or marginal products being exactly their averages, due to correlation between n and z and the nonlinearity of the production function.

- Idiosyncratic variables: Lucas-style approach again.
  - We make individual shocks uncorrelated. But how?
  - Principle of "neutrality": eliminating aggregate shocks we take to mean the *integration* over these shocks. This means:
    * For a general individual variable $y = g(\epsilon, z)$, where ϵ is an idiosyncratic and z an aggregate shock with a joint probability density $f(\epsilon, z)$, the individual variable without aggregate shocks becomes

      $$\tilde{y} = \int_z g(\epsilon, z) f(z|\epsilon) dz$$

      for each ϵ, with density $\int_z f(\epsilon, z) dz$.
    * In our case, $g(\epsilon, z) = \epsilon \in \{0, 1\}$, and the new prob. of employment is: $\pi_1 = \pi_{1g} + \pi_{1b}$.
    * That is, the employment process is *unaffected*.

INFINITE-HORIZON MODEL

We specify a model similar to Krusell/Smith (1998); we add stochastic discount factors and generate a realistic distribution of wealth. We also consider high risks of very persistent unemployment in recessions—long-term unemployment.

We eliminate cycles as of some point in time and track welfare for all agents as the economy transits to a steady state with constant prices. We compare this welfare to what would have resulted had cycles not been eliminated.

RESULTS

Steady state effect, baseline calibration: a gain of 0.138% (recall Lucas's 0.008%).

Steady state effect, calibration with long-term unemployed: a gain of 0.068%.

Transition experiment:

- An gain of about -0.01% on average across consumers. (The transition lowers the numbers!)

- Some agents gain substantially from eliminating cycles: those who are unemployed and have close to zero consumption (those at the borrowing constraint). They gain up to 2 full percentage points.

- The gain from eliminating cycles among the very poorest drops of very rapidly with wealth: just a tiny bit of wealth turns a gain from eliminating cycles into a loss.

- There are vanishingly few agents who gain full percentage points: agents save to stay away from zero consumption.

- If one excludes agents with the very lowest wealth, differences in welfare effects between groups are very small. They tend to be nonmonotone in wealth.

- The particular starting point (whether initial capital is high or low, and whether the initial aggregate shock is high or low) and ensuing transition path makes a difference for the results.

- If one eliminates cycles in an economy without a realistic wealth distribution, the (steady-state) costs of cycles fall by factor of about 7.

Welfare effects for different agents from eliminating cycles: The 2-state employment process

Initial state			Utility gain in percentage consumption					
			Employed agents (by wealth percentile)			Unemployed agents (by wealth percentile)		
z	$\bar{k}$	all agents	< 1	25-50	> 99	< 1	25-50	> 99
z_g	11.2	-0.008	-0.016	-0.007	-0.017	-0.016	-0.008	-0.017
z_g	12.3	-0.006	-0.010	-0.004	-0.021	-0.018	-0.004	-0.021
z_b	11.2	-0.010	-0.020	-0.012	0.014	-0.040	-0.013	0.014
z_b	12.3	-0.005	-0.008	-0.003	-0.031	-0.018	-0.003	-0.032

Welfare effects for different agents from eliminating cycles: The 3-state employment process											
		Utility gain in percentage consumption									
			Employed agents (by wealth percentile)			Short-term unemp. (by wealth percentile)			Long-term unemp. (by wealth percentile)		
Initial state		all									
z	$\bar{k}$	agents	< 1	25-50	> 99	< 1	25-50	> 99	< 1	25-50	> 99
z_g	11.2	-0.024	-0.059	-0.030	0.089	-0.150	-0.033	0.081	-0.340	-0.040	0.093
z_g	12.1	0.005	0.022	0.024	-0.241	-0.018	0.024	-0.225	-0.097	0.023	-0.252
z_b	11.2	-0.040	-0.038	-0.021	0.056	-0.075	-0.022	0.049	-1.019	-0.039	0.045
z_b	12.1	0.004	0.028	0.025	-0.261	0.012	0.025	-0.241	-0.148	0.018	-0.253

WHERE NEXT?

What is needed to increase the costs? Candidates:

1. Wage differences? Ríos-Rull has solved similar models and argues that wage differences are more important than employment status. More persistence?

2. Overlapping generations: finite lives/intergenerational transfers inoperative.

3. Housing: big lumpy asset with big fixed payments that causes trouble for unlucky consumers.

What else is needed? An actual model of individual and aggregate shocks.

- (in progress) Look at welfare costs of business cycles for different groups.

- Economic policy: redistribution.

- Look more carefully at how inequality is affected by business cycles.