1 Answer Keys for Homework 9

1.1 Exercise 1:

1. Each period the government will collect revenues issuing debt and through lump sum taxes on young (τ^y) and old (τ^o) individuals (where τ^y and τ^y are not necessarily equal). The revenues will be spent on defense and payment of last period's debt plus interest (at rate r_t) to the old. Notice that the interest rate is not necessarily constant and that $b_4 = 0$.

   Once you have the constraint from period 1 to 3, just solve for the debt levels and substitute to get the consolidated budget.

2. The young will earn income from working (wl). This will be spent on consumption, capital, debt and the payment of the lump sum tax (τ^y). When old, this individual will consume and pay taxes τ^o. The expenditures will financed via the payments of principal and interest of debt from the government and the principal and rental payments from capital. The interest from debt and capital should be the same (no arbitrage). Solving for "savings" in the young's budget and replacing it on the old's budget you'll get the consolidated budget. Just be careful since the young born in period 3 will never get to become old (so he or she will not save).

3. Using the optimality condition that MRS equals $R_t = (1 + r_t)$ together with the consolidated budget you can solve for consumption levels and savings. Notice that the savings will depend on income from working and the taxes paid when young and old: $s_{t+1} = b_{t+1} + k_{t+1} = (1 - \lambda)(w_t - \tau_t^y) + \frac{\lambda \tau_{t+1}^o}{(1+r_{t+1})}$

4. The firms will maximize profits: $AK^\alpha L^{1-\alpha} - wL - rK$. You can find the expressions for w and r on page 59.

5. The old is not taxed. To find the lump sum tax of the young just use the period t budget constraints of the government and the fact that $b_2 = b_3 = 0$. Notice that the first cohort of young pay defense expenditures (g_1) as well as what government owns to the cohort-zero old in terms of debt. To find out k_{t+1} as a function of k_t use the expression for savings derived in 3, the wage and interest rate obtained in 4 and the expressions of the tax to the young found out before. Finally, notice that while k_1 is given and k_4 is zero, k_2 will not equal k_3.

6. You can find the lump sum tax of the old using the same procedure as in 5. In this case, you will not be able to solve explicitly k_{t+1} as a function of k_t since r_{t+1} depends on k_{t+1} in a non-linear fashion.

7. With $\lambda = 0$, $K_{t+1} = w_t - \tau_t^y$. It is straightforward to show that capital in the second and third period is higher under the policy of $\tau_t^y = 0$ (the young have incentives to save). In terms of welfare, the idea is to compare the utility obtained under the two alternative policies. Since there is no

weight on the consumptions when young, you only need to compare the consumption when old. The cohort zero old (that living in period 1) is clearly better off when he is not being taxed. Now, for cohort i=1,2 we have $c^o_{i+1} = (1 + r_{i+1})K_{i+1} - \tau^o_{i+1}$.The result is ambiguous since capital is higher under $\tau^y_t = 0$, but the interest rate is lower plus the old are being taxed to finance defense.

8. The idea is to obtain the same levels of consumption, interest rates, wage rates and capital accumulation as in part 5 by taxing the old of generation 1 and 2, the young of generation 3 and issuing debt. To get the same level of consumption from all individuals you must be sure that they end up facing the same present value budget constraint as before. The only difference will be that gen.1 and 2 individuals will be taxed when old instead of when young. Thus, you should make sure that the present value of the new taxes equal the present value of the old ones. For example, the old of generation 1 will pay $\tau^o_2 = R_2(g_1 + b_1 R_1)$. In order to get the same consumption level for the young of generation 3 you need to set $\tau^y_3 = g_3$, as before. Finally, debt levels can be obtained from the government budget constraint in each period. You can check that this scheme will yield exactly the same levels as that of part 5.