

Homework 9

Consider a model with only three periods—after the third period, the economy ends, because of a sudden invasion of aliens. There is an old consumer alive in period 1, and a young consumer who survives to period 2 and then dies. In period 2, another consumer is born who survives to period 3. In period 3, there is yet another consumer born, but this consumer only gets one period of life, since the aliens take over after period 3.

The government starts with a debt to the private sector—a debt to the old in the first period—of $b_1 > 0$ units. The government needs to raise revenue from the consumers for the purpose of defense against alien invasions, of the amounts g_1 , g_2 , and g_3 . The government can raise lump-sum taxes on either young or old consumers in any period, and it can use borrowing, but as period 3 ends, it cannot have any remaining debt (they know in period 3 that the economy ends after that).

1. Describe the government's budget constraints in periods 1–3. Also derive the government's present-value budget constraint.
2. Describe each consumer's present-value constraint. The old consumer in period 1 has k_1 units of capital.
3. With the utility function $u(c_y, c_o) = c_y^\lambda c_o^{1-\lambda}$, solve each consumer's utility maximization problem: express their consumption levels and their savings as a function of the wages and interest rates and their present and future tax liabilities.
4. Assuming that the production function is $F(k, l) = Ak^\alpha l^{1-\alpha}$, derive expressions for the wages and returns to capital in the three periods. Use the information that in period t , the capital stock is k_t and the labor supply is 1.
5. Suppose that the government decides to make $b_2 = b_3 = 0$ and to tax the young, but not the old, in every period. Find the tax rates for each cohort, and find the capital accumulation equation. You should be able to express k_{t+1} as an explicit function of k_t for all three time periods.
6. Suppose that the government still maintains $b_2 = b_3 = 0$ but instead taxes the old, and not the young, in every period. Find the tax rates for each cohort, and find the capital accumulation equation. The capital accumulation equation will be an equation containing k_t and k_{t+1} , but in this case it will be impossible to solve for k_{t+1} as an explicit function of k_t .
7. Continuing on the last problem, now suppose that $\lambda = 0$, a special case, and solve explicitly for k_{t+1} as a function of k_t . (Notice that the young consumer in period 3 will be indifferent between consuming and saving; you may assume that he chooses to consume all his income.) With $\lambda = 0$, what policy—the present or the previous one—leads to the highest capital accumulation? How do the two policies compare in terms of welfare?

8. Show that there is a debt financing scheme together with taxation of the old of generations 1 and 2 (but no taxation of any young consumer nor of the initial old) that leads to exactly the same outcomes for all variables (interest rates, wage rates, consumption levels, capital accumulation) as did the first policy scheme, which had no taxes on the old but taxes on the young.