1 Answer Keys for Homework 6

2 Exercise 1:

1. Capital Accumulation Equation (with $\delta = 1$)

   -Write down the budget constraint that the young face.

   -Write down the budget constraint that the old face (notice that the old will get labor income as well)..

   -Use the above to obtain the individual's lifetime budget constraint (by substituting for "a_{t+1}"). Will the young and old face the same wage? Why or why not?.

   -State the maximization problem. It should read:

   $\max\limits_{c_o, c_y} \left\{ c_y^{\alpha} c_o^{1-\alpha} \right\}$ subject to: $w^y L^y + \frac{w^o L^o}{r_{t+1}} = c_y + \frac{c_o}{r_{t+1}}$.

   -Solve for c_y from the lifetime budget constraint (so you have an expression in two variables that you are maximizing over) and replace it inside the utility function.

   -Take First Order Conditions and solve for individual demand functions (c_y, c_o) as a function of parameters. Then determine the amount saved. You should obtain: $a_{t+1} = (1-\alpha) w^y L^y - \alpha \frac{w^o L^o}{r_{t+1}}$.

   -Find out w^y, w^o and r_{t+1} from the firm's maximization problem in periods t and $t+1$.

   -Use the fact that $N = 1$ and solve for the accumulation equation,

   $K_{t+1} = \frac{\beta L^y (1-\alpha)(1-\beta) A K^{\beta} L^{1-\beta}}{\beta L + (1-\beta)\alpha L^o}$. As a check, notice that if we set $L^o = 0$ and therefore $L = L^y$, you'll obtain the usual capital accumulation equation.

2. -Set $K_{t+1} = K_t = \bar{K}$ and use the capital accumulation equation to find the steady sate level of capital. You should get:

   $\overline{K} = \left[\frac{\beta L^y (1-\alpha)(1-\beta) A L^{1-\beta}}{\beta L + (1-\beta)\alpha L^o} \right]^{\frac{1}{\beta}}$

   -Replace $\overline{K}$ into the production function, the expressions for wages, interest rate and the consumption levels of each generation.

2.1 Exercise 2

1. Capital Accumulation Equation (with $\delta = 0$)

   -Write down the budget constraint that the young and the old face and determine the lifetime budget. Notice that you will have a different budget for each type.

   -State the maximization problem of a type i individual (where $i = 1, 2$). It should read:

$\max\limits_{c_o,c_y}\left\{c_y^{\alpha_i}c_o^{1-\alpha_i}\right\}$ subject to: $w^iL^i = c_y + \frac{c_o}{(1+r_{t+1})}$

-Take First Order Conditions and solve for individual demand functions (c_y, c_o) as a function of parameters. Then determine the amount saved by each type. You should obtain: $a_{t+1}^i = (1-\alpha_i)\,w^iL^i$.

-Define $L = L^1N^1 + L^2N^2$. Determine the wages faced by each type. Notice that the labor provided by type 1 and 2 individuals can be perfectly substituted in production.

-Total capital is the sum of the total savings of type 1 plus those of type2 individuals. Find the accumulation equation You should get: $K_{t+1} = (1-\beta)AK^{\beta}L^{-\beta}\left[L^1N^1(1-\alpha_1) + L^2N^2(1-\alpha_2)\right]$.

-Set $K_{t+1} = K_t = \bar{K}$ and use the capital accumulation equation to find the steady sate level of capital. You should get: $\bar{K} = \left[\frac{(1-\beta)A}{L^{\beta}}\left[L^1N^1(1-\alpha_1) + L^2N^2(1-\alpha_2)\right]\right]^{\frac{1}{\beta}}$.

-Replace $\overline{K}$ into the production function, the expressions for wages and interest rate. Also solve for the steady state values of consumption and savings of each type.

2. -Find the fraction of total labor income accruing to type 1 people. You should get: $\frac{L^1N^1}{L^1N^1+L^2N^2}$.

   -Using this describe the parameters affecting labor income inequality and who is better off when these parameters change.

   -Find the fraction of total capital income accruing to type 2 people. You should get: $\frac{L^2N^{22}(1-\alpha_2)}{L^1N^1(1-\alpha_1)+L^2N^{22}(1-\alpha_2)}$.

   -Using this describe the parameters affecting labor income inequality and who is better off when these parameters change.

3. Now,$Y = AK^{\beta}L_s^{\gamma}L_u^{1-\beta-\gamma}$

   -They are complements.

   -Solve the firm's maximization problem: $\max\{AK^{\beta}L_s^{\gamma}L_u^{1-\beta-\gamma} - w_sL_s - w_uL_u\}$. Determine w_s and w_u and simplify. You should find that the relative wage between skill and unskilled labor will depend on only four parameters.

   -Solve the maximization problem following the same steps than in part 2 and solve for the aggregate accumulation equation. Set $K_{t+1} = K_t = \bar{K}$ and use the capital accumulation equation to find the steady sate level of capital. You should get: $\bar{K} = \left[AL_s^{\gamma}L_u^{1-\beta-\gamma}\left[(1-\alpha_1)\gamma + (1-\alpha_2)(1-\beta-\gamma)\right]\right]^{\frac{1}{\beta}}$.

   -Replace $\overline{K}$ into the production function, the expressions for wages (of each type) and interest rate. Also solve for the steady state values of consumption and savings of each type.

   -Use the fact that total labor income of the skilled is $\bar{w}_sL_s$ and that of the unskilled is $\bar{w}_uL_u$ to determine the distribution of labor income.

-Use the fact that total capital income of the skilled is $(1+\bar{r})\bar{K}^s$ and that of the unskilled is $(1+\bar{r})\bar{K}^u$ to determine the distribution of capital income ($\bar{K}^s$ and $\bar{K}^u$ are the savings of skilled and unskilled types).