

Homework 6

1. (Capital accumulation, continued) Consumers have the utility function $u(c_y, c_o) = c_y^\alpha c_o^{1-\alpha}$ and are endowed with L_y units of labor when young and L_o units of labor when old. Assume that there is only one consumer per generation ($N = 1$). With a production function $AK^\beta L^{1-\beta}$ and 100% depreciation of capital, do the following:
 - (a) Determine the capital accumulation equation: the equation giving K_{t+1} as a function of K_t .
 - (b) Find the steady state.
2. (Capital accumulation and inequality) Suppose that there are two different kinds of consumers in every generation, type 1 and type 2. Type 1 consumers have preferences $u(c_y, c_o) = c_y^{\alpha_1} c_o^{1-\alpha_1}$ and type 2 consumers have preferences $u(c_y, c_o) = c_y^{\alpha_2} c_o^{1-\alpha_2}$, with $\alpha_1 \neq \alpha_2$. Type 1 consumers are endowed with L_1 units of labor when young and nothing when old, and type 2 consumers are endowed with L_2 units of labor when young and nothing when old. There are N_1 consumers of type 1 and N_2 consumers of type 2 in each generation. The production function is $AK^\beta L^{1-\beta}$.
 - (a) Find the accumulation equation for total capital and find the steady state.
 - (b) In reality, both labor income and capital income are very unevenly distributed. Describe what parameters in this model influence inequality in these two income sources: find the fraction of total labor income accruing to type 1 people and the fraction of total capital income accruing to type 2 people. Explain.
 - (c) Suppose now that the two kinds of consumers have different kinds of labor inputs, rather than simply different amounts. In particular, assume that type 1 is endowed with “skilled” labor, and type 2 with “unskilled” labor (with the same amounts as above). The production function now is $AK^\beta L_s^\gamma L_u^{1-\beta-\gamma}$, where L_s is the total amount of skilled labor input and L_u the total amount of unskilled labor input and where both β and γ are between zero and one. Are the two kinds of labor substitutes or complements? What parameter values will influence the relative wage between skilled and unskilled labor? Find the steady state and the long-run labor and capital income distributions for this economy.