Homework 5

(Capital accumulation)

1. For a utility function $u(c_y, c_o) = c_y^{0.5} c_o^{0.5}$ and a production function $10K^{0.2}L^{0.8}$, do the following:

   (a) Determine the capital accumulation equation: the equation giving K_{t+1} as a function of K_t.

   (b) Find the steady state.

   (c) Suppose that initial output is 10% of steady state output (because initially, there is not much capital in the economy). How many periods does it take before output is within 10% of the steady-state output level?

   (d) Change the production function to $10K^{0.8}L^{0.2}$ and answer the previous question again.

2. If the utility function is $\log c_y + B \log c_o$ and the production function is $A_t K^{\beta} L^{1-\beta}$, where $A_{t+1} = (1 + a)A_t$, what is the long-run output growth rate and what is the long-run relation between K_t and A_t?

3. Consider a steady state of the kind of economy we considered in class, without technological change. Now suppose the government considers allowing a significant increase in immigration: without this increase, the total population would remain constant over time, but with the new immigration, the population of workers would once-and-for-all double, beginning immediately. Assume that the immigrants arrive in period 1 and are all young and will be able to work in that period. Next period (and all periods after that), the population is at a higher level: the immigrants and their offspring have as many children as the initial citizens.

   (a) What would the long-run effect of the potential increase in immigration be on output per capita?

   (b) How would it affect the welfare of the initial old?

   (c) How would it affect the welfare of the initial young (those workers who are already citizens)?

   (d) How would it affect future generations?

4. Suppose that there is physical depreciation of capital between periods t and $t + 1$: if K_{t+1} is saved in period t, what will remain after production is merely $(1 - \delta)K_{t+1}$. Let, as before, the production function be Cobb-Douglas.

   (a) Describe the budget constraints of the consumer: notice that for every unit saved, the return is now $1 - \delta + r_{t+1}$.

   (b) Find the capital accumulation equation and the steady state, assuming that preferences are Cobb-Douglas—how is the steady state affected by the rate of depreciation?

(c) Define gross investment as $K_{t+1} - (1 - \delta)K_t$ and net investment as $K_{t+1} - K_t$. What are the steady-state levels of gross and net investment?

(d) Given that, on average, depreciation rates for capital are about 10% per year, and given that we interpret a period in our model as about 30 years, what is a realistic value for δ in the model?

(e) Given that the capital share is 30% and that the annual real interest rate, net of depreciation, in the U.S. has been around 4%, we can use the steady state of this model to estimate how large the capital-output ratio is. What is the implied ratio of the capital stock to annual output?

(f) What is the implied ratio of consumption to output?