1 Answer Keys for Homework 4

1.1 Exercise 1:

1.1.1 Part (a)

-Write down the budget constraint that the young face.

-Write down the budget constraint that the old face.

-Use the above to to obtain the individual's lifetime budget constraint (by substituting for "a_{t+1}").

-State the maximization problem. It should read:

$\max_{c_o, c_y} \left\{ c_y^{\gamma} c_o^{1-\gamma} \right\}$ subject to: $w = c_y + \frac{c_o}{1+r_{t+1}}$

-Solve for c_y from the lifetime budget constraint, replace it inside the utility function, take First Order Conditions and solve for individual demand functions (c_y, c_o) as a function of parameters. You should obtain the following:

$c_y = \gamma w$

$c_o = (1-\gamma)(1+r)w$

$s = w - c_y = (1-\gamma)w$

1.1.2 Part (b)

If there are "N" consumers in each generation, then total savings among the young must equal "N" times the amount saved by the representative young consumer. Likewise, total savings among the old will equal "N" times the amount saved by the representative old consumer. National Savings is just the sum of the former:

$S_t = Ns_{y,t} + Ns_{o,t}$

It is also, by definition:

$S_t \equiv Y_t - Nc_{y,t} - Nc_{o,t}$

If the income of individuals equals total output we have:

$Y_t = Nw_t + r_t Na_t = Nw_t + r_t K_t$

By now you should be able to obtain the first equation by Combining the latter two.

After rearranging, you should get:

$S_t \equiv N(w_t - c_{y,t}) + rK_t - Nc_{o,t}$

Investment is by definition:

$I_t = K_{t+1} - K_t$

Finally, use the individual budget constraints (of the young and the old at "t") and the fact that "$Na_s = K_s, \forall s$" to complete the proof.

1.1.3 Part (c)

To answer this, notice that in an open economy people are allowed to pursue investment projects abroad and at the same time domestic investment projects may be financed by foreign capital.

1.2 Exercise 2:

Part (a)

Firms maximize their profits in a competitive environment. They choose how much labor to hire and capital to rent depending on current wage and rental rates. Formally:

$$\max_{l_t,k_t} \{y_t - w_t l_t - r_t k_t\}$$
$$subject\ to : y_t = k_t^{\beta} l_t^{1-\beta}$$

Or, if we replace for y_t :

$$\max_{l_t,k_t} \left\{ k_t^{\beta} l_t^{1-\beta} - w_t l_t - r_t k_t \right\}$$

After taking first order conditions you should arrive to the following:

$r_t = \beta k_t^{\beta-1} l_t^{1-\beta} = \beta(k_t/l_t)^{\beta-1}$

$w_t = (1-\beta) k_t^{\beta} l_t^{-\beta} = (1-\beta)(k_t/l_t)^{\beta}$

Solving for the capital labor ratio answers the question.

Another way of answering the question (easier but less insightful) is by starting with the two marginal product conditions from the book, page58 (it can be proven that they can be derived from the above first order conditions). Just replace "Y_t" with "$K_t^{\beta} L_t^{1-\beta}$" and you will be able to answer the question.

1.2.1 Part (b)

To answer this just use the wage and rental rate equations you obtained in part (a)

1.2.2 Part (c)

Use the same two equations but this time compute the new population number by increasing it 5% for every one of the four years.

1.2.3 Part (d)

Again use the marginal productivity of labor and of capital equations to find out. Take into account that if capital increases at a higher rate than labor does then the capital-labor ratio will rise.

1.2.4 Part (e)

Replace "Y_t" with "$AK_t^{\beta} L_t^{1-\beta}$" in the marginal productivity equations and find out.

1.2.5 Part (f)

Both countries share the same technology "A_t" but may differ in their total inputs "K, L".

If wages are going down in one country, say country "D" (domestic). Then, formally we have:

$(1-\beta)A_{t+1}(K_{D,t+1}/L_{D,t+1})^\beta = w_{D,t+1} < w_{D,t} = (1-\beta)A_t(K_{D,t}/L_{D,t})^\beta$

$\Rightarrow \frac{A_{t+1}}{A_t} < \left[\frac{(K_{D,t}/L_{D,t})}{(K_{D,t+1}/L_{D,t+1})}\right]^\beta$

If we assume that technology is improving or at least staying the same, then

$1 \leq \frac{A_{t+1}}{A_t} < \left[\frac{(K_{D,t}/L_{D,t})}{(K_{D,t+1}/L_{D,t+1})}\right]^\beta \Rightarrow 1 < \frac{(K_{D,t}/L_{D,t})}{(K_{D,t+1}/L_{D,t+1})} \Rightarrow \frac{(K_{D,t+1}/L_{D,t+1})}{(K_{D,t}/L_{D,t})} < 1$

This means that in country "D" the capital labor ratio must be falling.

If you allow for negative technological change. This would invalidate the previous result.

But in any case, the above implies:

$\left[\frac{A_{t+1}}{A_t}\right]^{\frac{1}{\beta}} < \left[\frac{(K_{D,t}/L_{D,t})}{(K_{D,t+1}/L_{D,t+1})}\right] \Rightarrow$

$$\left[\frac{(K_{D,+1t}/L_{D,t+1})}{(K_{D,t}/L_{D,t})}\right] < \left[\frac{A_{t+1}}{A_t}\right]^{\frac{-1}{\beta}}$$

If wages are going up in the other country, say country "F" (Foreign). Then, formally we have:

$(1-\beta)A_{t+1}(K_{F,t+1}/L_{F,t+1})^\beta = w_{F,t+1} > w_{F,t} = (1-\beta)A_t(K_{F,t}/L_{F,t})^\beta$

$\Rightarrow \frac{A_{t+1}}{A_t} > \left[\frac{(K_{F,t}/L_{F,t})}{(K_{F,t+1}/L_{F,t+1})}\right]^\beta \Rightarrow$

$$\frac{(K_{F,t+1}/L_{F,t+1})}{(K_{F,t}/L_{F,t})} > \left[\frac{A_{t+1}}{A_t}\right]^{\frac{-1}{\beta}}$$

Interest rates are going up in both counties, thus:

$\beta A_{t+1}(K_{D,t+1}/L_{D,t+1})^{\beta-1} = r_{D,t+1} > r_{D,t} = A_t(K_{D,t}/L_{D,t})^{\beta-1}$

$\Rightarrow \frac{A_{t+1}}{A_t} > \left[\frac{(K_{D,t}/L_{D,t})}{(K_{D,t+1}/L_{D,t+1})}\right]^{\beta-1}$

$\Rightarrow \frac{A_{t+1}}{A_t} > \left[\frac{(K_{D,t+1}/L_{D,t+1})}{(K_{D,t}/L_{D,t})}\right]^{1-\beta} \Rightarrow$

$$\left[\frac{A_{t+1}}{A_t}\right]^{\frac{1}{1-\beta}} > \frac{(K_{D,t+1}/L_{D,t+1})}{(K_{D,t}/L_{D,t})} \quad and \quad \left[\frac{A_{t+1}}{A_t}\right]^{\frac{1}{1-\beta}} > \frac{(K_{F,t+1}/L_{F,t+1})}{(K_{F,t}/L_{F,t})}$$

Combining ALL the previous results we have that:

$$\left[\frac{A_{t+1}}{A_t}\right]^{\frac{1}{1-\beta}} > \frac{(K_{F,t+1}/L_{F,t+1})}{(K_{F,t}/L_{F,t})} > \left[\frac{A_{t+1}}{A_t}\right]^{\frac{-1}{\beta}} > \frac{(K_{D,+1t}/L_{D,t+1})}{(K_{D,t}/L_{D,t})}$$

Thus, what we can say for sure is that the relative capital-labor rate between countries "F" and "D" is increasing.

We cannot say anything about the growth rate of capital-labor rates within countries, since we have no information about the rate of technological change. However, if we assume that technological change cannot be declining, then we know that the capital-labor rate in country "D" is declining.

1.2.6 Part (g)

If borders had been open then price differences (wages and capital rental rates) would have converged. If wages were lower in one country, migration would take place. Then the supply of labor would decrease in the low wage country and increase in the high wage one. This would lower the wage rate in the high wage country and increase it in the low wage country until they reached convergence. The same idea applies to capital. If capital could move freely and costlessly across the borders then capital rental rates would also converge.

Thus, in order to tell whether prices increased or decreased we should know which country had the higher/lower wage and rental rates when the borders were opened.