Homework 4

1. (National saving)

   (a) For a utility function $u(c_y, c_o) = c_y^{\gamma} c_o^{1-\gamma}$, and wages and interest rates w_t and r_t in period t, derive, first, the lifetime budget constraint and, second, the optimal consumption-savings behavior.

   (b) If there are N consumers in each generations, what is the total amount of saving in period t among the young? What is the total amount of saving among the old? What is the total amount of saving in the whole population? Show that in a closed economy, if the income of the individuals equals total output, this must equal the total increase in the capital stock in the economy: total domestic investment.

   (c) If the economy is open, will total national saving necessarily equal total domestic investment? If yes, explain; if no, explain both why the former might be larger and why the latter might be larger.

2. (Profit maximization and determination of prices)

   (a) If the production function is Cobb-Douglas, $K^{\beta} L^{1-\beta}$, what does profit-maximizing behavior imply for the optimal choice of the capital-labor ratio? (I.e., how does the latter have to be related to input prices—the wage rate and the interest rate?)

   (b) Assuming that $\beta = 0.5$, if the overall amount of labor in the population is 7 and the amount of capital is 4, what do prices have to be to ensure that all labor and all capital is employed by profit-maximizing firms?

   (c) If the labor force grows by 5% per time period, and capital does not grow, what will prices be after 4 years? Explain the intuition for this finding.

   (d) If capital grows at a higher rate than the rate at which labor grows, what will then happen to prices? Explain the intuition.

   (e) Now suppose that the production function is $AK^{\beta} L^{1-\beta}$, where A is technology. What happens to prices when A grows and K/L stays the same?

   (f) Consider two countries with the stated production function (allowing the technology parameter A); we know the production function is the same in the two countries because they are neighbors and are at an equal level of development. Suppose the countries do not allow labor or capital to flow across the borders. Suppose, furthermore, that any technology growth that occurs is known to be common across the countries, whereas input growth rates may differ across countries. If we observe interest rates moving up in both countries and we observe wage rates moving down in one country and up in the other, what can be concluded about the growth of capital, the growth of labor, and the growth of technology in the two countries? If you cannot say if they went up or down, say what you can about the capital-labor ratios, or about the relative capital-labor ratios across the two countries.

(g) Still within the context of these two countries, if borders had been open and labor and/or capital could have moved freely and costlessly, what would have happened to prices in the two economies? Can we tell whether interest rates and wage rates would have gone up or down?