1 Answer Keys for Homework 3

1.1 Exercise 1:

1.1.1 Part (a)

Marginal Utility of "c_y" $= A\frac{1}{2}c_y^{-\frac{1}{2}}c_o^{\frac{1}{2}}$

Marginal Utility of "c_o" $= A\frac{1}{2}c_y^{\frac{1}{2}}c_o^{-\frac{1}{2}}$

Marginal Rate of Substitution $= -\frac{A\frac{1}{2}c_y^{-\frac{1}{2}}c_o^{\frac{1}{2}}}{A\frac{1}{2}c_y^{\frac{1}{2}}c_o^{-\frac{1}{2}}} = -\frac{c_o}{c_y}$

Indifference curve at $(U = 1)$: $c_o = \frac{A}{c_y}$

1.1.2 Part (b)

Marginal Utility of "c_y" $= \lambda\frac{1}{c_y}$

Marginal Utility of "c_o" $= (1-\lambda)\frac{1}{c_o}$

Marginal Rate of Substitution $= -\frac{\lambda}{1-\lambda}\frac{c_o}{c_y}$

Indifference curve at $(U = 1)$: $c_o = e^{\frac{1}{1-\lambda}}c_y^{-\frac{\lambda}{1-\lambda}}$

1.1.3 Part (c)

Marginal Utility of "c_y" $= 3$

Marginal Utility of "c_o" $= 1$

Marginal Rate of Substitution $= -3$

Indifference curve at $(U = 1)$: $c_o = 1 - 3c_y$

1.1.4 Part (d)

Marginal Utility of "c_y" $= c_o^2$

Marginal Utility of "c_o" $= 2c_yc_o$

Marginal Rate of Substitution $= -\frac{c_o}{2c_y}$

Indifference curve at $(U = 1)$: $c_o = c_y^{-\frac{1}{2}}$

1.1.5 Part (e)

Marginal Utility of "c_y" $= 2e^{-2c_y - c_o}$

Marginal Utility of "c_o" $= e^{-2c_y - c_o}$

Marginal Rate of Substitution $= -2$

Indifference curve at ($U = -0.01$): $c_o = -\ln(0.5) - 2c_y$

1.2 Exercise 2:

1.2.1 Part (a)

-Write down the budget constraint that the young face.

-Write down the budget constraint that the old face.

-Use the above to to obtain the individual's lifetime budget constraint (by substituting for "a_{t+1}").

-State the maximization problem. It should read:

$\max_{c_o, c_y} \left\{ c_y^{\alpha} c_o^{1-\alpha} \right\}$ subject to: $w = c_y + \frac{c_o}{1+r_{t+1}}$

-Solve for c_y from the lifetime budget constraint, replace it inside the utility function, take First Order Conditions and solve for individual demand functions (c_y, c_o) as a function of parameters. You should obtain the following:

$c_y = \alpha w$

$c_o = (1-\alpha)(1+r)w$

Savings $= a = w - \lambda w$

1.2.2 Part (b)

-Write down the budget constraint that the young face.

-Write down the budget constraint that the old face.

-Use the above to to obtain the individual's lifetime budget constraint (by substituting for "a_{t+1}").

-State the maximization problem. It should read:

$\max_{c_o, c_y} \{\lambda \ln(c_y) + (1-\lambda)\ln(c_o)\}$ subject to: $w_y + \frac{w_o}{1+r} = c_y + \frac{c_o}{1+r}$

-Solve for c_y from the lifetime budget constraint, replace it inside the utility function, take First Order Conditions and solve for individual demand functions (c_y, c_o) as a function of parameters. You should obtain the following:

$c_y = \lambda \left[w_y + \frac{w_o}{1+r} \right]$

$c_o = (1-\lambda)\left[(1+r)w_y + w_o\right]$

Savings $= a = w_y - \lambda \left[w_y + \frac{w_o}{1+r} \right]$

Give specific values so that $a < 0$.

1.2.3 Part (c)

-Write down the budget constraint that the young face.

-Write down the budget constraint that the old face.

-You can use the individual's lifetime budget constraint from part (b).

-State the maximization problem. It should read:

$\max\limits_{c_o,c_y}\left\{-e^{-2c_y-c_o}\right\}$ subject to: $w_y+\frac{w_o}{1+r}=c_y+\frac{c_o}{1+r}$

-Solve for c_y from the lifetime budget constraint, replace it inside the utility function. Note that by taking first order conditions you may not be able to obtain an interior solution. Depending on whether the interest rate is high enough or not individuals may want to consume only when young or only when old. Check the indifference curves from Exercise (1) to understand why.

$c_y=\left[w_y+\frac{w_o}{1+r}\right]$ if $r<1$ and zero otherwise

$c_o=(1+r)w_y+w_o$ if $r>1$ and zero otherwise

Savings $=w_y$ if $r>1$ and $-\frac{w_o}{1+r}$ otherwise

Give specific values so that $a<0$.

1.3 Exercise 3

1.3.1 Part (a)

An increase in the interest rate will increase the future value of income. Thus, there are incentives to increase consumption in the future. However, depending on how you draw the indifference curves you may obtain ANY of the desired effects.

Intuitively, higher income induces higher consumption both when old and when young and higher interest rates induce higher consumption when old. This means that both the substitution and income effect should reinforce each other towards higher future consumption. You may still be able to create an example where future consumption actually decreases but such a reaction to an increase in the interest rate is very unusual and not the type of reaction we see in the data.

You will not be able to construct an example where the consumer is worse off. This is simply because an increase in the interest rate will increase the choice set of the individual. In other words, with the new budget constraint the individual can still access any possible consumption-savings decision he could take before the increase in the interest rate.

1.3.2 Part (b)

Normally, increasing the interest rate should induce consumers to increase future consumption (pure substitution effect due to a decrease in the price of future consumption). This would increase savings. Thus you need the income effect to decrease savings by more than the increase by substitution. To obtain the opposite result when doing the graph, just make sure that the income effect does not counterbalance the substitution effect.

To obtain a decrease in consumption when old, you would need an extremely negative income effect. The best way to obtain this is by assuming that the individual receives almost all income when old. This way an increase in the interest rate decreases the present value of income.