Homework 3

1. (Marginal rates of substitution) For the following utility functions, compute the marginal rate of substitution (MRS) function, plot the indifference curves (with c_y on the x axis and c_o on the y axis), and indicate MRS is in the graph. Also determine, for each case, whether the marginal utility of each good is decreasing and whether the MRS function is decreasing.

   (a)

   $$u(c_y, c_o) = A\sqrt{c_y}\sqrt{c_o},$$

   with A being a positive constant.

   (b)

   $$u(c_y, c_o) = \lambda \log c_y + (1 - \lambda) \log c_o,$$

   with λ being a constant in between zero and one.

   (c)

   $$u(c_y, c_o) = 3c_y + c_o.$$

   (d)

   $$u(c_y, c_o) = c_y c_o^2.$$

   (e)

   $$u(c_y, c_o) = -e^{-2c_y - c_o}.$$

2. (Utility-maximizing savings decisions)

   Solve the consumers consumption-savings decisions, both algebraically and graphically, in the following three cases:

   (a) Utility function: $u(c_y, c_o) = c_y^\alpha c_o^{1-\alpha}$, with $0 < \alpha < 1$. Income: w when young, nothing when old. Prices: interest rate r. Suppose that $\alpha = 0.5$ so that, in some sense, you care equally about consumption when young and when old. Under what condition on the interest rate r does the consumer choose "perfect consumption smoothing", that is, to consume the same when young and when old? Explain why the consumer does not perfectly consumption smooth if the interest rate does not satisfy this condition.

   (b) Utility function: $u(c_y, c_o) = \lambda \log c_y + (1 - \lambda) \log c_o$, with $0 < \lambda < 1$. Income: w_y when young and w_o when old. Prices: interest rate r. Give some specific numerical values for λ, w_y, w_o, and r that are such that the consumer borrows (not lends) when young.

(c) Utility function: $u(c_y, c_o) = -e^{-2c_y - c_o}$. Income: w_y when young and w_o when old. Prices: interest rate r.

3. (Borrowing and lending) Suppose we do not specify a particular utility function but simply assume that the MRS is decreasing (so that the indifference curves are convex). Also, remember from micro that the indifference curves cannot cross.

   (a) Assume that the consumer does not have income when old, but only when young. Show graphically that an increase in the interest rate may increase saving or decrease saving, depending on the particular shape of the indifference curve. Explain how it is possible that savings may decrease: use the concepts of income and substitution effects from Intermediate Micro. Is it possible that consumption when old will decrease when interest rates increase? Explain, again using income and substitution effects. Is the consumer necessarily better off when the interest rate goes up?

   (b) Assume now that the consumer has income both when young and when old. Show with a sequence of graphical examples that consumption when young and when old can go either up or down when the interest rate goes up, and that the consumer may be made better off or worse off, depending on preferences and on his income combination. For each example, explain the intuition verbally.