Homework 2

1. (Long-run growth and cross-country differences)

   (a) Figure 1.11 of the book shows 7 economies and their 1960 GDP along with their average annual growth rates over the period 1960–1992. Your task is to *assume* that each of these economies will grow at the same rate as before and then to calculate its respective prospected GDP for year 2024. Plot the results. Is the distribution of world income becoming more even or more uneven? Is the internal ranking changing? Is the assumption you were asked to make realistic? Why or why not?

   (b) Figure 1.9 in the book ranks countries according to GDP growth rates over 1960–1994. One hypothesis about long-run growth is *convergence*: countries that are initially relatively poor grow faster than those that are initially rich. What would this hypothesis imply for the ranking of incomes of the countries in the figure (note: this ranking is not shown in the figure!)? When you consider the specific countries involved, does the hypothesis seem to be correct? Defend your answer.

   (c) The convergence hypothesis can, as is argued in Chapter 3, be derived simply from the assumption that there are decreasing returns to capital: a rich country has a lot of capital, and therefore the returns from increasing the capital stock further is low, leading rich countries to save at low rates and thus not growing much richer. A poor country, on the other hand, would have high returns to capital and therefore save at high rates and become rich fast.

   Alternatively, the convergence hypothesis is based on an assumption about technology rather than saving in capital: to catch up technology-wise is much easier than to develop new technology from scratch, because you can copy technology by looking at what is available in the rich countries—copying is easier than invention. Therefore, the leaders cannot improve technology-wise as fast as those that are behind.

   Use either or both of these ideas to interpret Figure 1.10 in the book: offer an explanation in terms of capital or technology accumulation for what has happened to Hong Kong, South Korea, Peru, China, and the Ivory Coast. These countries all started far behind the world leaders.

2. (Life-cycle budgets)

   (a) Consider the life-cycle model of Chapter 2. If the young earn income w_{yt} and the old also earn income, of the amount w_{ot} in period t, what will the life-time budget constraint be? Let r_{t+1} be the net real interest rate between periods t and $t+1$.

   (b) Plot the life-time budget constraint for this case, as in Figure 2.4. Depict two possible consumption choices, one that implies that the young agent is a net saver, and one that implies that he is a net borrower, or dissaver. Describe the amount (dis)saved in the picture for each case.

(c) If wages are $w_{yt} = 10$ and $w_{ot} = 5$ at all points in time, if the net interest rate is 1 (100%), if each generation decides to consume the same amount when young as when old, and if the size of each generation is 50, show that the total capital stock will have to equal 250/3 in every period.

(d) Now change the model of question (2a) slightly: assume that the agent of generation t when old gives a bequest, b_{t+1}, to the next generation (that the next generation receives in period $t + 1$). How do the budget constraints of the young and old change? Assuming that generation t consists of N_t agents, how will the total (economy-wide) capital stock in period $t + 1$, K_{t+1}, depend on the savings and bequest decisions of the agents alive in period t?

(e) If a model time period is roughly half of an adult life, what is a realistic value for r_{t+1}, the real interest rate?

(f) The real interest rate between periods t and $t + 1$ can be interpreted as a relative price, involving consumption good at t and consumption good at $t + 1$ (which, of course, can be viewed as two different goods). Make the statement more precise and interpret.

(g) We will now further complicate the model of question (2a): suppose now that an individual of generation t lives for three periods: youth (period t), mid-life ($t+1$), and old age ($t + 2$). The individual receives wealth as an inheritance from his "parents" of generation $t - 1$ in the period in which he is middle-aged; call this amount b_{t+1}. Use c_{yt} and w_{yt} for consumption and labor income when young, c_{mt+1} and w_{mt+1} for the corresponding items when middle-aged, and c_{ot+2} and w_{ot+2} for the corresponding items when old. Moreover, use b_{t+2} for the amount bequeathed by this individual to his heir. Finally, let a_{yt+1} be what the agent saves when young and a_{mt+2} be what he saves when middle-aged. Write down the agent's budget constraints in each period, and then write down the life-time budget constraint. Finally, if all generations have the same structure (live for three periods and have children when middle-aged, so that there are three generations alive at each point in time), and if each generation is of size N, how can total capital be expressed as a function of the savings and bequest choices of the different generations in period t?