1 Answer Keys for Homeworks 11-12

1.1 Exercise 1:

There is an easy way to solve this without the use of differentiation. Just make sure you have the proper definitions of the following:

-Rate of growth of money stock (you may call it "μ").

-Inflation Rate (you may call it "π").

-Real Demand for money.

Use these three plus the fact that the Real Demand for money is constant over time (lets say, between "t" and "$t+1$") and the market clearing condition for the money market to complete the proof.

1.2 Exercise 2:

The solution of this exercise will be needed to solve for exercises (3) and (4) so be very careful when setting up the problem:

-Write down the budget constraint that the young face.

-Write down the budget constraint that the old face.

-Write down the equation relating nominal interest rate, inflation rate and real interest rate.

-Use the above to to obtain the individual's lifetime budget constraint (by substituting for "a_{t+1}").

-State the maximization problem. It should read:

$\max\limits_{c_o, c_y, m_{t+1}} c_y^{\theta_1} c_o^{\theta_2} (\frac{m_{t+1}}{p_t})^{1-\theta_1-\theta_2}$ subject to $w = c_y + \frac{c_o}{1+r_{t+1}} + \frac{m_{t+1}}{p_t} \frac{i_{t+1}}{1+i_{t+1}}$ (substitute for c_y, for example, from the budget to have an expression in two variables that you are maximizing over).

-Take First Order Conditions and solve for individual demand functions $(c_y, c_o, \frac{m_{t+1}}{p})$ as a function of parameters. For consumption you should obtain the following:

$c_y = \theta_1 w$

$c_o = \theta_2 (1+r) w$

-Obtain the real demand for money and answer the question .

1.3 Exercise 3:

Since the individual maximization problem is the same, the demand functions found in (2) may be used here.

-Use the demand functions and the budget constraint of the young to solve for the individual's demand of assets "a_{t+1}".

-Since we are looking for a steady state with no monetary growth, you can use the result in exercise (1) to obtain the inflation rate (why?).

-Use the inflation rate above to replace the nominal interest rate in terms of the real interest rate.

-Knowing the production function, you can obtain both "w" and "r" in terms of parameters.

-Replace these into the asset (capital) demand equation.

-Solve for the steady state capital. (It will NOT be equal to the one in the growth chapter).

-Solve for the steady state price level by using the demand function for money.

The steady state capital level is:

$k_{ss} = \left\{ \frac{\left(\frac{\beta}{1-\beta}\right)+1-\theta_1-\theta_2}{A\beta\theta_2} \right\}^{\frac{-1}{1-\beta}}$

1.4 Exercise 4

The only differences between this and exercise (3) are that we are using a different production function and we are not necessarily on a steady state. We can still use the results from exercise (2) since the individual maximization problem is the same:

-Rewrite the demand functions for capital and money.

-Do not use the result in exercise (1) this time but do replace "i" in terms of "π" and "r".

-Knowing the NEW production function obtain "w" and "r" in terms of parameters.

-Obtain the new money and asset (capital) demand functions by replacing "w" and "r" for their new values.

-In the money demand function, notice that:

$\frac{1+i}{i} = \frac{1}{i} + 1 = \frac{1}{r+\pi(1+r)} + 1$

-write the inflation rate "π" in terms of prices.

-Solve for "p_{t+1}" as a function of parameters and "p_t".

You should get:

$$p_{t+1} = \frac{1}{1+A} \frac{1}{\frac{1}{p_t} - B(1-\theta_1-\theta_2)/m},$$

which is a function that starts in 0, has slope $1/(1+A)$ there, is convex, then crosses the 45 degree line at some point $p^\star$ and goes to infinity as the price goes to $\frac{m}{B(1-\theta_1-\theta_2)}$. What solutions are there? Inspecting the graph, one solution is that the price is $p^\star$ always. Then it looks like there are solutions where the price goes to zero, if the initial price is any price between 0 and $p^\star$, but those solutions are not really solutions: they imply that the real demand for money eventually becomes higher than the wage of the current young, which is infeasible! And the initial price cannot start above $p^\star$ either: then there is no positive solution for prices after some periods. In conclusion: the price must be $p^\star$ from the beginning of time and then stay there—nothing else works.

1.5 Exercise 5:

This question is easily anwered by taking into account the effect of the actions taken by the government on prices. A complete answer would also take into account what happens to people who have debts/credits in nominal and/or real terms.

1.6 Exercise 6

For the second question you should take into account how equal amounts of money affect people with different money holding and/or different nominal debit/credit positions. Finally, for the last question you should first define the government policy on nominal credit/debit positions (what happens if the government also adds a zero to all debts/credits and what happens if it does not?) and then answer accordingly.