1 Answer Keys for Homework 10

1.1 Exercise 1

1. You need to find out how much will each individual save (use the $MRS = 1 + r_{t+1}$): $K_{t+1} = N_t(1-\alpha)B$. Aggregate capital grows at the rate of $(1+n)$. Capital per capita is at steady state after one period.

2. Each young will pay sB and each old will receive $(1+n)sB$ due to population growth. After some algebra you can find that $c^y = \frac{\alpha B}{(1+A)}\left[(1+A)(1-s)+(1+n)s\right]$ and $c^o = (1-\alpha)B\left[(1+A)(1-s)+(1+n)s\right]$. Using the individual budget constraint you can solve for capital holdings and find out that in steady state there is less capital than in a system with no social security. Aggregate capital still increases at the same rate as population does.

3. The initial old clearly gain with the introduction of the social security system since they receive a transfer without having to pay anything when young. For the other generations we have to compare the utility levels with and without the system. Replacing the consumption levels in each case (found in parts 1 and 2 respectively) and simplifying, you can see that the system will be beneficial only if $n > A$. That is, as long as population grows at a higher rate than the productivity of capital every generation will be better off with a SS system than without it. Give some intuition on this. Notice that the answer is independent of B and α.

4. The consumption of young and old will now depend on s_t and s_{t+1}. In order to not make cohorts 1,2,... and so on worse off with the SS system than without it we need $s_{t+1} > (1+A)s_t > (1+A)^t s_1$(the first inequality comes from substituting consumption in both cases on the utility function and comparing them). Over time this proportion is getting bigger by a factor of $(1+A)$. Since the fraction s is restricted to be less than unity in any period of time (since we cannot take from an individual more than he gets as labor income) this system will be unfeasible.

5. With no population growth and under system (i), from period t and on consumption levels are $c^y = \alpha B$ and $c^o = (1-\alpha)B(1+A)$. Under the second system, the old in t is paid what he was promised and hence it consumes the same as any other old previously did. Any individual born in t and after will consume the same as under system 1 (ie, c^y and c^o). In order to pay for the transfer to the old living in period t, the government will borrow $b_{t+1} = sB$. Notice that in order to pay for this without taxing anyone, it will have to issue $b_{t+2} = (1+A)b_{t+1}$. One period later, $b_{t+3} = (1+A)b_{t+2} = (1+A)^2 b_{t+1}$. The debt level will be increasing at the rate $(1+A)$ each period. Since capital holdings cannot be negative, this second system becomes unfeasible at some point. Be sure that you can find out how to get this result by using individual's consumption function and their constraints. An intermediary system that could work would be to borrow

sB from the young and tax the old in a lump sum manner by AsB units. In this way, the government can pay back the interest from debt and it will be borrowing a constant amount every period.

6. Baby Boom: The only affected by the baby boom is the old living in period t. He will be benefited by the temporary increase in population since instead of receiving sB units, he'll get $sB(1+n)$ as a transfer. Since n goes back to zero after that, the other individuals in the economy do not change their behavior.

1.2 Exercise 2

1. If the weight on c_y is zero in the utility function ($\alpha = 0$), then everything is consumed when old. Thus, whatever is not paid by the young to social security is saved in the form of assets. That is:

$K_{t+1} = N_t a_{t+1} = (1-s)w_t N_t = (1-s)(1-\beta)AK_t^{\beta}N_t^{1-\beta}$. Calculate the steady state of per capita capital using this equation.

As for the interest rate, we know it equals the marginal productivity of next periods capital:

$$r_{t+1} = \beta AK_{t+1}^{\beta-1}N_{t+1}^{1-\beta} = \beta A\left[(1-s)(1-\beta)AK_t^{\beta}N_t^{-\beta}\right]^{\beta-1}\left[(1+n)N_t\right]^{1-\beta}$$

$$r_{t+1} = \frac{1}{(1-s)^{1-\beta}}\beta A\left[(1-\beta)AK_t^{\beta}N_t^{-\beta}\right]^{\beta-1}\left[(1+n)N_t\right]^{1-\beta}$$

In this setting, to answer the question about the interest rate it suffices to realize that if no social security system exists then $s = 0$.