Homework 10

1. Consider an economy where consumers have preferences $u(c_y, c_o) = c_y^\alpha c_o^{1-\alpha}$ and where the production function is $F(K, L) = AK + BL$, with $A > 0$ and $B > 0$: that is, the marginal products of the inputs are not decreasing, and the two inputs are not complements—the marginal product of labor is independent of how much capital there is, and vice versa.

   (a) If the population initially has N_0 old consumers and every subsequent cohort is n percent larger than the previous one, what path will capital accumulation follow? What will the steady state look like, and how long will it take to reach the steady state?

   (b) Suppose now that the government decides to enact social security at time 1, giving the old of generation 0 a *fraction* s of the total labor income of the young, split up equally among all the old consumers. Suppose moreover that the government promises that the social security system will stay intact in the future: in every period, the young will pay the same fraction s of their income in social security taxes, and receive a social security transfer when they are old. How do capital accumulation and the steady state change compared to when there was no social security system?

   (c) Determine who gains and who loses from the introduction of social security as outlined above. That is, calculate the utility of each cohort with and without social security. Does the answer depend on the values for A, B, α, and n? Explain.

   (d) Assume now, for simplicity, that $n = 0$, and suppose that the government, instead of promising to keep the system intact forever, increases the fraction s over time: at time 1, the fraction is s_1, and at time 2, it is s_2, and so on, where $s_2 > s_1$, and similarly for the future. How much do the s_t's need to increase in order to not make cohorts 1, 2, and so on worse off with social security than without it? What will happen to the social security system over time?

   (e) Suppose that $n = 0$ and that the fraction s has been constant over time until a time t when the government decides that it will want to eliminate social security. Compare the following two systems: (i) simply do not pay the old at t anything; and (ii) pay the old what they were promised, but do it by borrowing from the young, not by taxing them; then repay the debt at $t + 1$, and borrow from the young at $t + 1$ to repay the debt; at $t + 2$, repay the debt to the old, and borrow from the young, and so on. Do both systems "work"? In particular, is it possible to use the second system, given that nobody is ever taxed? Can you think of some intermediate system with debt and taxes that would work?

   (f) Analyze a "baby boom": analyze what will happen to the utility of different cohorts if s is constant over time, n is zero for a while, and then becomes high for one period, and then goes back to zero again and stays there forever. What is the effect on the utility of different cohorts of the social security system in this case?

2. Consider a version of the growth model with: Cobb-Douglas preferences with a zero weight on consumption when young, Cobb-Douglas production with a capital share of β, 100% depreciation, income only when young, and a population that is growing at a rate of n percent per period. Assume that there is a social security scheme in place, and that it works as in the previous question: each period, every young consumer pays a *fraction* s of his current income in social security taxes and the government transfers these revenues directly to the current old. Determine the capital accumulation equation and the steady state. In the previous question, the interest rate was left unchanged by the introduction of social security. Is that true here as well? If not, how is the interest rate affected? Explain.