

Homework 1

1. (Growth, cycles, and compounding)

- (a) Suppose the output of a country grows at $x\%$ for five years, and then at $x/2\%$ for the next five years, then for five years at $x\%$ again, and so on. Another country grows at $x\%$ the first year and then alternates *every year* between this value and $x/2\%$. In the 10th year, how do the output levels of these two countries compare?
- (b) What is the average, long-run growth rate of each of the two countries? Would it be better, in some sense, or worse, for these countries to have grown at an even pace, assuming this even pace had been the long-run average? That is, would it have been better to avoid cycles? Explain.
- (c) Two countries start out producing the same GDP per capita in 2001, but one of the countries produces twice as much as the other 10 years later. How much faster did this country grow on average (in percent) per year?

2. (Production functions)

- (a) Show, using a Cobb-Douglas production function with capital share θ strictly between 0 and 1, that
 - i. there are decreasing returns to capital and decreasing returns to labor;
 - ii. the marginal productivity of labor is increasing in capital, and vice versa;
 - iii. the average productivity of labor is increasing in capital, and vice versa;
 - iv. there are constant returns to scale by showing that if all the inputs increase by x percent, output increases by x percent;
 - v. if the firm purchases inputs so as to set each input price equal to its marginal product, it will make zero profits; and
 - vi. the capital and labor shares of income do not depend on the inputs.
- (b) Suppose the production function is $AK + BL$ (not Cobb-Douglas), where A and B are positive constants. Which of the properties above will hold for this function and which will not?

3. (Types of technological change) Give two concrete examples of disembodied technological change in production (other than those mentioned in class).

4. (Growth accounting)

- (a) A country exhibits the following official data for the period 1950–2000: the average growth rate of capital was 3%, the average growth rate of output was 2%, the average growth rate of labor input was 0.5%, and the capital share of income was 30% every year. Using growth accounting techniques, what percentage of output growth over the period was due to growth in labor input, in capital input, and in technology?

- (b) Now someone claims, for the same country, that output was not well measured—the price increases were not just capturing inflation, but also increased quality of output, and the official accounts did not take quality increase of output into account at all. The claim, more precisely, is that quality growth was 1% per year. Redo the growth accounting with this knowledge.
- (c) Someone else claims that output was not mismeasured but capital was: capital quality increased, to the extent that capital input growth was underestimated by 2 percentage points in the official statistics. Redo the growth accounting with this perspective instead.
- (d) Finally, someone claims that there has been a long-run decrease in utilization of both labor and capital, and that this change in utilization has gone unobserved by the national accountants. The claim is that capital stands idle more often, and that labor takes leisure on the job to a much larger extent (puts in much less effort); based on survey information, the estimate is that both capital and labor were utilized only half as much in 2000 as in 1950. Use this information to adjust the national accounts.