# FINAL EXAM, ECO 209H, FALL 2001

**Note: the 20% "new" questions are #s 1 and 4**

1. (10 points) A country exhibits the following official data for the period 1900–2000: the average growth rate of capital was 9%, the average growth rate of output was 7%, the average growth rate of labor input was 1%, and the capital share of income was 25% every year.

   (a) Using growth accounting techniques, what percentage of output growth over the period was due to growth in labor input, in capital input, and in technology?

   (b) Detailed analysis of the century shows that both output and labor input growth were fairly constant over the whole time period, whereas capital growth was much higher in the second half of the period than in the first. Based on this information, did the role of residually measured technology growth increase or decrease from the first to the second half of the century? Explain.

2. (10 points) Solve the consumers consumption-savings decisions, both algebraically and graphically, in the following case. Utility function: $u(c_y, c_o) = \lambda \log c_y + (1-\lambda) \log c_o$, with $0 < \lambda < 1$. Income: $w_y$ when young and $w_o$ when old. Prices: interest rate $r$. Give some specific numerical values for $\lambda$, $w_y$, $w_o$, and $r$ that are such that the consumer borrows (not lends) when young.

3. (15 points) Suppose that there are two different kinds of consumers in every generation, type 1 and type 2. Type 1 consumers have preferences $u(c_y, c_o) = c_y^{\alpha_1} c_o^{1-\alpha_1}$ and type 2 consumers have preferences $u(c_y, c_o) = c_y^{\alpha_2} c_o^{1-\alpha_2}$, with $\alpha_1 \neq \alpha_2$. Type 1 consumers are endowed with $L_1$ units of labor when young and nothing when old, and type 2 consumers are endowed with $L_2$ units of labor when young and nothing when old. There are $N_1$ consumers of type 1 and $N_2$ consumers of type 2 in each generation. The production function is $AK^{\beta}L^{1-\beta}$. Find the accumulation equation for total capital and find the steady state.

4. (10 points) Consider a growth model where preferences are given by $u(c_y, c_o) = -(e^{-c_y} + e^{-c_o})$ and the production function by $F(K, L) = AK + BL$. Find the steady-state capital stock. (If you have trouble dealing with functions like $e^x$, note that $\frac{d}{dx}e^x = e^x$ and that if $e^x = y$, then $x = \log y$, where log means "natural log", or ln.)

5. (10 points) Consider a steady state of the kind of economy we considered in class, without technological change. Now suppose the government considers allowing a significant increase in immigration: without this increase, the total population would remain constant over time, but with the new immigration, the population of workers would once-and-for-all double, beginning immediately. Assume that the immigrants arrive in period 1 and are all young and will be able to work in that period. Next period (and all periods after that), the population is at a higher level: the immigrants and their offspring have as many children as the initial citizens.

   (a) What would the long-run effect of the potential increase in immigration be on output per capita?

   (b) How would it affect the welfare of the initial old?

6. (5 points) Recall the growth accounting exercises from Chapter 1, and the homework associated to it. Multifactor productivity, $A$ in our model, as measured using growth accounting, is procyclical: it correlates positively with the output. Is this evidence that technology shocks are the fundamental cause of business cycles? In your answer, comment on the possible role of labor hoarding and capacity utilization.

7. (10 points) Using 1990 as base year, compute real GDP for 1990–1992 based on the following price and quantity information: there are two goods, and good 1 cost 100, 100, and 110 over the period whereas good 2 cost 100, 110, and 110, whereas their corresponding quantities were 20, 22, and 24 for good 1 and 20, 25, and 30 for good 2. Also, compute a chain-weighted index for the period. For each index, compute the GDP deflator.

8. (15 points) Derive an individual's demand for real money balances under the following assumptions: (i) the individual can save in real assets at a net real rate of $r$; (ii) the individual has income $w$ when young and nothing when old; and (iii) the individual's utility function is $u(c_y, c_o, m/p) = c_y^{\theta_1} c_o^{\theta_2} (m/p)^{1-\theta_1-\theta_2}$, with $\theta_1$, $\theta_2$, and $\theta_1 + \theta_2$ strictly between 0 and 1. Does the demand for money depend on the real interest rate, the nominal interest rate, or both?

9. (5 points) If the government prints money and buys government bonds in an open-market operation, is there a tax on anybody? If it prints money and gives it for free to the population in equal amounts per person, is there a tax on anybody? If the government decides by decree to increase the money stock by adding a zero to every note and coin (a nickel becomes 50 cents, a $20-bill becomes a $200-bill, etc.), is there a tax on anybody? Explain, in all cases.

10. (10 points) Using the formulas for, and the ideas behind, the IS and LM curves in the textbook, show graphically what the effect is of

    (a) a sudden increase in the economy-wide price level; and

    (b) an increase in government taxation of the young agents

    on the real interest rate and on output.