Typos Answer Keys for Homework 3

Exercise 2 a):

Perfect consumption smoothing means that consumption when old and consumption when young are the same.
Under what conditions on *1+r,* do consumers choose perfect consumption smoothing?.

$$C_o = C_y,$$
$$(1-\alpha)[1+r]w = \alpha w,$$
$$[1+r] = \frac{\alpha}{1-\alpha}.$$

When C_o and C_y are the same, the marginal rate of substitution (MRS) between C_o and C_y is equal to $\alpha/(1-\alpha)$.

$$MRS = \frac{\partial U / C_y}{\partial U / C_o} = \frac{\alpha}{(1-\alpha)} \frac{C_y^{\alpha-1} C_o^{1-\alpha}}{C_y^{\alpha} C_o^{-\alpha}} = \frac{\alpha}{(1-\alpha)} \left(\frac{C_o}{C_y} \right).$$

If $(1+r)=[\alpha/(1-\alpha)]$ then the optimal decision is perfect consumption smoothing.

Typos Answer Keys for Homework 5

Exercise 2:

The capital accumulation equation is:

$$K_{t+1} = \frac{B}{B+1}(1-\beta) A_t K_t^{\beta} L^{1-\beta}.$$

Agent's maximization problem:

$$Max.U(C_y, C_o) = \log C_y + B \log C_o$$
$$st$$
$$C_y + a' = w \rightarrow C_y = w - a',$$
$$C_o = (1+r)a'.$$

$$
\begin{aligned}
&Max.\log(w-a') + B\log((1+r)a') \\
&FOC: \\
&\frac{dU}{da'} = -\frac{1}{w-a'} + \frac{B(1+r)}{(1+r)a'} = 0. \\
&a' = \frac{B}{1+B}w.
\end{aligned}
$$

Firm's maximization problem

$$
\begin{aligned}
&Max.\pi = A_t K^{\beta} L^{1-\beta} - wL - rK, \\
&FOC: \\
&w = A_t(1-\beta)K^{\beta}L^{-\beta}, \\
&r = A_t \beta K^{\beta-1}L^{1-b}.
\end{aligned}
$$

Capital accumulation equation:

$$
\begin{aligned}
&k_{t+1} = \frac{B}{1+B}A_t(1-\beta)k_t^{\beta}, \\
&k_t = \frac{K_t}{L}.
\end{aligned}
$$

We know that the productivity factor *A*, grows at a constant rate *(1+a)*. We have to find the long run output growth rate $(1+g_y)$:

$$
\begin{aligned}
&k_{t+1} = \frac{B}{1+B}A_t(1-\beta)k_t^{\beta}, \\
&k_{t+2} = \frac{B}{1+B}A_{t+1}(1-\beta)k_{t+1}^{\beta} = \frac{B}{1+B}(1+a)A_t(1-\beta)k_{t+1}^{\beta}, \\
&\frac{k_{t+2}}{k_{t+1}} = (1+a)\left(\frac{k_{t+1}}{k_t}\right)^{\beta}, \\
&(1+g_k) = (1+a)(1+g_k)^{b}, \\
&(1+g_k) = (1+a)^{\frac{1}{1-\beta}}.
\end{aligned}
$$

In the long run, capital per capita and technology grow at different rates

$$
\begin{aligned}
Y_{t+1} &= A_{t+1}K_{t+1}^{\beta}L^{1-\beta} = A_t(1+a)K_{t+1}^{\beta}L^{1-\beta}, \\
\frac{Y_{t+1}}{L} &= A_t(1+a)\left(\frac{K_{t+1}}{L}\right)^{\beta}, \\
y_{t+1} &= A_t(1+a)k_{t+1}^{\beta}, \\
y_t &= A_t k_t^{\beta}, \\
\frac{y_{t+1}}{y_t} &= (1+a)\left(\frac{k_{t+1}}{k_t}\right)^{\beta}, \\
(1+g_y) &= (1+a)(1+g_k)^{\beta}, \\
(1+g_y) &= (1+a)\left[(1+a)^{\frac{1}{1-\beta}}\right]^{\beta} = (1+a)(1+a)^{\frac{\beta}{1-\beta}}, \\
(1+g_y) &= (1+a)^{1+\frac{\beta}{1-\beta}} = (1+a)^{\frac{1}{1-\beta}}.
\end{aligned}
$$

In the long run, output per capita and capital per capita grow at the same rate, but technology grows at a different rate.

$$1+g_y = 1+g_k \neq 1+a.$$