

### Midterm Exam, 11/8–2001

1. (a) (15 points) A country exhibits the following official data for the period 1950–2000: the average growth rate of capital was 3%, the average growth rate of output was 2%, the average growth rate of labor input was 0.5%, and the capital share of income was 30% every year. Using growth accounting techniques, what percentage of output growth over the period was due to growth in labor input, in capital input, and in technology?
  - (b) (4 points) Both inputs and outputs may be mismeasured in the official statistics. Describe three different kinds of mismeasurements that would lead to an overestimate of the role of capital accumulation for growth. You do not need to provide specific numbers.
2. Consumers have the utility function  $u(c_y, c_o) = c_y^\alpha c_o^{1-\alpha}$  and are endowed with  $L_y$  units of labor when young and  $L_o$  units of labor when old. Assume that there is only one consumer per generation ( $N = 1$ ). With a production function  $AK^\beta L^{1-\beta}$  and 100% depreciation of capital, do the following:
  - (a) (20 points) Determine the capital accumulation equation: the equation giving  $K_{t+1}$  as a function of  $K_t$ . Show how you derive it.
  - (b) (5 points) Find the steady state, and show how you derive it.
3. Consider a model with only three periods—after the third period, the economy ends, because of a sudden invasion of aliens. There is an old consumer alive in period 1, and a young consumer who survives to period 2 and then dies. In period 2, another consumer is born who survives to period 3. In period 3, there is yet another consumer born, but this consumer only gets one period of life, since the aliens take over after period 3.

The government starts with a debt to the private sector—a debt to the old in the first period—of  $b_1 > 0$  units. The government needs to raise revenue from the consumers for the purpose of defense against alien invasions, of the amounts  $g_1$ ,  $g_2$ , and  $g_3$ . The government can raise lump-sum taxes on either young or old consumers in any period, and it can use borrowing, but as period 3 ends, it cannot have any remaining debt (they know in period 3 that the economy ends after that).

- (a) (10 points) Describe the government's budget constraints in periods 1–3. Also derive the government's present-value budget constraint.
- (b) (3 points) Describe each consumer's present-value constraint. The old consumer in period 1 has  $k_1$  units of capital.
- (c) (10 points) With the utility function  $u(c_y, c_o) = c_y^\lambda c_o^{1-\lambda}$ , solve each consumer's utility maximization problem: express their consumption levels and their savings as a function of the wages and interest rates and their present and future tax liabilities.

- (d) (5 points) Assuming that the production function is  $F(k, l) = Ak^\alpha l^{1-\alpha}$ , derive expressions for the wages and returns to capital in the three periods. Use the information that in period  $t$ , the capital stock is  $k_t$  and the labor supply is 1.
- (e) (12 points) Suppose that the government sets  $b_2 = b_3 = 0$  and taxes the old, and not the young, in every period. Find the tax rates for each cohort, and find the capital accumulation equation. The capital accumulation equation will be an equation containing  $k_t$  and  $k_{t+1}$ , but it will be impossible to solve for  $k_{t+1}$  as an explicit function of  $k_t$ .

4. Consider the following data for the last 100 years for country 1 and country 2. Country 1 has had an average inflation rate of 3% per year and country 2's inflation rate has been 5% per year. Nominal returns on 1-year government bonds have been 5% in country 1 and 9% in country 2. In each country, returns on bonds have equalled net returns on capital accumulation (by net we mean returns after subtraction for depreciation). Output per capita has been roughly constant over time in the two countries, with country 1 at about half the output per capita of country 2. Depreciation rates in each country are believed to be 1% per year. The two countries have been entirely closed to trade with the outside world.

You are to assume that the countries are in a steady state associated with the growth model in class. Production functions are Cobb-Douglas in each country, and the share of income paid to labor is constant and equal to 50% in each country. Preferences are Cobb-Douglas and income is only received when young. Interpret a period in the model as 30 years.

- (a) (2 points) What is the total real return on government bonds in each country over a model period (i.e., over 30 years, including compounding of returns)?
- (b) (2 points) What is the depreciation rate in each country over a model period?
- (c) (4 points) Use the previous information to compute the marginal product of capital in country 1 for a model period.
- (d) (4 points) Use the information about the marginal product of capital to compute the capital/output ratio in country 1 for a model period.
- (e) (4 points) Is it possible to tell whether country 2 has higher output per capita than country 1 because they saved more, because they had a higher level of technology, or both?