Econ 483 Sept. 8, 1998
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Fall 1998

Problem Set 1

1. A *partition* of a set E is a countable disjointed collection of subsets whose union is E. It is called a *finite partition* if it has only finitely many elements. Let $\{A, B, C\}$ be a partition of E. Describe the $\sigma-$algebra generated by this partition.

2. Show that a *discrete* probability space cannot contain an infinite sequence $A_1, A_2, ...$ of independent events each of probability 1/2.

3. Two prisoners out of three (call them prisoners A, B, C) have been chosen at random to be shot at dawn. To maintain the prisoner's interest, the results will be announced at the last minute. Prisoner A asks a guard, "Which of B and C will be shot? I know one of them will be, so knowing which one gives me no information about myself." The guard agrees, and informs A that C is to be shot. Now A is quite relieved, believing that his chance of survival to be 50% (either he or B will survive).

   Is A living a fool's paradise? Calculate the conditional probability that A gets shot at dawn, given the guard's statement. Is there sufficient information in the statement of the problem to answer the question?

4. Suppose there is an experiment with three possible events A, B, C with respective probabilities p_A, p_B, p_C (with $p_A + p_B + p_C = 1$ of course). If this experiment is conducted repeatedly, with each experiment being an independent trial under identical conditions, what is the probability that event A occurs before event B does?

5. A particular diagnostic test for a disease is 95% accurate on both those that do and do not have the disease. If 0.5% of the population actually has the disease, what is the conditional probability that a randomly chosen individual actually is infected, given that the test indicates that (s)he does?

6. Suppose three events A, B, C affect each other in the following way:

$$P(B|C) = 1/2, \ P(B|C^c) = 1/3, \ P(A|B) = 1/2, \ P(A|B^c) = 1/3$$

Furthermore, suppose that the event A, conditional on B or B^c, is independent of C; i.e:

$$P(A|B \cap C) = P(A|B) \quad \text{and} \quad P(A|B^c \cap C) = P(A|B^c)$$

Calculate the conditional probabilities $P(A|C)$ and $P(A|C^c)$.

7. Let $(-, \mathcal{F}, P)$ be a probability space, where $- = (0, 1]$, $\mathcal{F} = \sigma(\mathcal{I})$ where $\mathcal{I}$ is the class of subintervals of $-$, and $P(\omega \in (a, b]) = b - a$ if $0 < a \leq b \leq 1$. Find the cumulative distribution functions and (discrete or continuous) probability density functions for the following random variables:

   (a) $X(\omega) = \ln(\omega/(1 - \omega))$

   (b) $Y(\omega) = \sum_{i=0}^{\infty} I[\omega > (1 - p)p^i], p \in (0, 1)$ $\quad$ ($I[A] = 1$ if A is true, 0 otherwise).

   (c) $Z(w) = \text{tangent}(\pi(\omega - 1/2))$

   (d) $W(\omega) = \text{sign}(\omega - 1/2) \cdot \ln(2 \cdot \min(\omega, 1 - \omega))$