Lecture 8

Relaxing the assumptions: Zero-Beta CAPM, Taxation, and Borrowing-Lending constraints

AIM OF LECTURE 8

- Relax some of the assumptions underlying the Capital Asset Pricing Model (CAPM)

8.1 ZERO-BETA CAPM

Why no risk-free asset?
- inflation uncertainty
- credit rationing

The zero-beta CAPM is due to Black (1972). We will do a "heuristic derivation."[1]

Even if there is no risk-free asset we may draw a line tangent to the market portfolio. This is done below in Figure 8.1. The intercept is not the return on the risk-free asset, but the expected return on something else, say portfolio z: $E[\tilde{R}_z]$.

This line is similar to the Capital-Market Line in Lecture 7, just that R_f is replaced by $E[\tilde{R}_z]$. Since the derivative of the frontier is the same as in Lecture 7, the CAPM relation will hold but with R_f replaced by $E[\tilde{R}_z]$:

$$E[\tilde{R}_j] - E[\tilde{R}_z] = \beta_j.(E[\tilde{R}_m] - E[\tilde{R}_z]) \qquad \text{(Zero-Beta CAPM)}$$

where

$$\beta_j \equiv \mathrm{COV}(\tilde{R}_j,\tilde{R}_m)/\mathrm{VAR}(\tilde{R}_m)$$

<u>Which portfolio is *z*?</u>

Set j=z then the above equation becomes $0 = \beta_z.(E[\tilde{R}_m] - E[\tilde{R}_z])$ and thereby we must have

$$\beta_z = 0$$

that is

$$\mathrm{COV}(\tilde{R}_z,\tilde{R}_m) = 0$$

So z is a portfolio which return is uncorrelated with the market portfolio, z is the *zero-beta portfolio*.

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[1] Full derivation is in Copeland and Weston pp. 205-208. The principle is the same as for the standard CAPM, i.e. finding the slope of the efficient frontier.

Figure 8.1

μ □ $\text{slope} = \dfrac{E[R_m]-E[R_z]}{\sigma_m}$

μ_m □ $\text{slope} = \dfrac{\partial E[R_m]/\partial a}{\partial \sigma_m/\partial a}$

μ_{GMV}

$E[R_z]$

σ_m σ

<u>Is z unique?</u>

Well, in fact, any portfolio which is uncorrelated with the market portfolio must have the same expected return (since such a porfolio contains the same (zero) amount of market risk).

So any i such that $COV(\tilde{R}_i,\tilde{R}_m) = 0$ must have $E[\tilde{R}_i] = E[\tilde{R}_z]$.

Or the other way around any portfolio with $E[\tilde{R}_i] = E[\tilde{R}_z]$ must have zero beta. All these portfolios lie on the dashed line in the Figure 8.2, below.

Figure 8.2

μ slope = $\dfrac{E[R_m]-E[R_z]}{\sigma_m}$

μ_m slope = $\dfrac{\partial E[R_m]/\partial a}{\partial \sigma_m/\partial a}$

μ_{GMV}

$E[R_z]$ ↙Z

σ_z σ_m σ

<u>But</u>, there is only *one* zero-beta portfolio which has got minimum variance, i.e. portfolio *z* in the figure above.

<u>Where is *z* on the frontier?</u>

The zero-beta portfolio is always on the <u>inefficient</u> part of the portfolio frontier (as in the figure above).

<u>Fund separation in the Zero-Beta CAPM?</u>

Remember the *spanning property* of the frontier portfolios: any portfolio on the frontier can be obtained as a linear combination of only two portfolios on the frontier. Therefore <u>we have two-fund separation</u> even without the risk-free asset. (But **not**, of course, two-fund monetary separation).

Equilibrium in the Zero-Beta CAPM world

Figure 8.3

μ

μII

II

μm

□ m

μI

μGMV

I

$E[R_Z]$

σI σm σII σ

N.B. Points on the straight line not possible portfolios (unless at point *m*).

When may the Zero-Beta CAPM fail?

An important assumption behind the Zero-Beta CAPM is that short-sales are possible. To obtain zero-beta portfolios we typically would have to short sell some assets. If there are short-sales constraints the Zero-Beta CAPM fails to hold.

8.2 DIFFERENT BORROWING AND LENDING RATES

Suppose borrowing rate is greater than the lending rate: $R_B > R_L$.

Figure 8.4

μ

μB □ borrowing portfolio, B

μL □ lending portfolio, L

μGMV

RL

σL σB σ

All lenders hold portfolio *L*.
All borrowers hold portfolio *B*.
Others hold portfolio between *L* and *B*, according to preferences.

8.3 NON-MARKETABLE ASSETS

Example: human capital.

The standard CAPM breaks down when investors are "forced" to hold *non-marketable* assets. Mayers (1972) does derive a CAPM-type relationship of security prices

$$E[\tilde{R}_j - R_f] = \hat{\beta}_j . E[\tilde{R}_m - R_f] \qquad \text{(7.26 in C\&W)}$$

where

$$\hat{\beta}_j \equiv \frac{V_m \mathrm{COV}(\tilde{R}_j, \tilde{R}_m) + \mathrm{COV}(\tilde{R}_j, \tilde{R}_H)}{V_m \sigma_m^2 + \mathrm{COV}(\tilde{R}_m, \tilde{R}_H)}$$

$\tilde{R}_H$ = monetary return (i.e. in pound sterling) on all nonmarketable assets,

V_m = current value of all marketable assets

The return (in percent) on an asset *j* depends on its covariability with the marketable porflolio *and* with the nonmarketable portfolio, and the covariability between the marketable and the nonmarketable portfolio.

We should note:

(i) Individuals hold different portfolios of risky assets depending on their holdings of the non-marketable assets.

(ii) The pricing relation (7.26) is independent of individuals' preferences (as in the stadard CAPM).

(iii) The appropriate measure of risk is still the covariance.

8.4 TAXES

Differential taxation of capital gains and dividends. We may be tempted to think that shares would be valued according to their *dividend yield* as well as the level of market risk.

$$E[\tilde{R}_j - R_f] = \beta_j . E[\tilde{R}_m - R_f] + F(DY_j, DY_m, \tau)$$

where

DY_j = dividend yield on asset *j*

DY_m = dividend yield on the market portfolio

τ = vector of various taxes

and *F* is a function.

However, we then ignore a number of aspects, like optimal capital structure of the firm. For example, in cases where there dividends are taxed more than capital gains the firm could instead of paying out dividends, buying its own shares of the same amount, which implies that the investor in the end only pays the capital gains tax. This is a rather complex issue, beyond the scope of this course, but it is important to be aware that differential tax treatment may not cause the CAPM to break down.

8.5 TRANSACTION COSTS

If there are "large" transactions costs then investors may limit the number of assets in their portfolio, then we cannot have a simple CAPM relationship for returns. Realistic?

8.6 NEXT TIME

Next time we will go though an alternative asset-pricing model, the Arbitrage Pricing Theory (APT).

REFERENCES

Copeland, Thomas E., and J. Fred Weston, *Financial Theory and Corporate Policy*, Addison-Wesley, chapter 7, part G, in particular: pp. 205-208, 209-210, 211-212.
Huang, Chi-fu, and Robert H. Litzenberger, *Foundations for Financial Economics*, North-Holland, pp. 70-72.