Economics and Incentives in Remanufacturing

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Abstract

With environmental concerns and legislative pressures mounting, more firms worldwide and in the US are remanufacturing their products in order to recover value and reduce waste. This paper examines the situations where existence of third part remanufacturers reduces the incentives from the original equipment manufacturer (OEM) to remanufacture the product.

A two-period two-player undiscounted profit maximization model is formulated. Consumer demand is incorporated into this. A linear demand function yields closed form solutions for the optimal prices set by the two players. Using this some insights are presented about the behavior of the OEM.

Keywords: remanufacturing, green manufacturing, repairable goods, Stackelberg games

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1 Introduction

1.1 Environmental Concerns and Remanufacturing

Each year approximately xx billion tons of waste is generated in the US. The number of landfills available has drastically reduced since the 70s. The available landfills are not adequate to take up the increase in the waste generated each year. New landfills are difficult to create due to increasing opposition from communities and environmental groups.

States are legislating to ensure that more products are environmentally friendly, that is they are easily recyclable or reusable. In Europe, it is now mandatory for many manufacturers to take back their product after its useful life.

Due to increased environmental consciousness among consumers, communities and lawmakers, many firms are examining their products in order to facilitate recycling and remanufacturing. Remanufacturing is recreation of a new unit by reusing many of the original components from an old unit. The parts may be used as is, or after rework. Recycling is reusing the material of the product through operations that result in destruction of the form, for example, by grinding or melting.

In many products, the material costs are a small portion of the total cost. Most of the value is added during manufacture as the product components take up their final form. In many units, components may be useful although the old unit has no more use. Remanufacturing these products retains a larger component of value than recycling, and can create benefit if it used fewer resources than needed to manufacture the same component. Remanufacturing inherits the benefits of recycling, like reducing waste.

1.2 Remanufacturing In Industry

Remanufacturing is prevalent in a variety of industries – diesel locomotive engines, presses, automobiles, PCs, single use cameras, toner cartridges, and many more. The total size of the remanufacturing industry is estimated to be about $53 billion from 73,000 firms[1]. A single model cannot apply to all industries. In this paper, we will focus on those items which are small and sell in large volumes, like single use cameras, toner cartridges, etc. This model is also valid for those industries where there is significant third party remanufacturing activity.

The original equipment manufacturer (OEM) is usually not the only one to remanufacture the product. Small operators in local markets will also remanufacture the product[2].

Third party remanufacturers may not have the economies of scale that the OEM has in remanufacturing. On the other hand, they may use technologies that the OEM cannot use efficiently. The OEM may often set up dedicated facilities for the remanufacturing chain. In many cases, remanufacturing is one of the many operations that the local shop does. Thus, both transaction costs and remanufacturing costs may be higher for the local shop. Since remanufacturing usually recaptures a large part of the original value of the product, it is possible for the local shop to operate.

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1 "The Remanufacturing Industry: Hidden Giant" Boston University Study, 1994 funded by Argonne National Laboratory. The study covered 11,000 firms in 8 industry sectors (automotive, compressors, electrical apparatus, machinery, office furniture, tires, toner cartridges and valves).

2 For example, local shops refill inkjet cartridges, or sell refill kits, while Xerox and Canon (the originaal manufacturers) also refill and sell the cartridges. Smaller shops put new film in used single-use cameras and resell them. Kodak and Fuji also remanufacture them.

1.3 Incentives for OEM and Benefits to Society

The OEM has to balance the benefits of remanufacturing against the loss of used units to local shops. For mature products where the total sale volume grows slowly, this diversion may be a significant loss of revenue for the OEM. Thus, the OEM may prefer to sell fewer items in order to reduce the input to local remanufacturers.

There are other benefits of remanufacturing for the OEM, for instance availability of parts of discontinued products. However, there are significant costs of reverse logistics for the used items. Not all items are returned for remanufacture. Due to obsolescence, not all used parts may be economically remanufacturable.

In most cases, remanufacturing the product reduces the environmental burden, and thus provides a net benefit to society. In addition, in a standard economic sense, since remanufacturing costs less, the consequent price reduction increases the quantity consumed, and hence increases the total surplus to the consumers and manufacturers.

1.4 Consumer Behavior

Consumer behavior is modeled using demand functions for the items. The demand for the original item and the remanufactured item is the same. However, it is not the same for the OEM's and the local remanufacturer's items. This may be due to brand effects, availability and access to the products, or other reasons.

The demand function captures the effects of price competition between the two players. Thus any change in the price by one player affects the quantities sold by both players.

This model also assumes that the items have a natural return stream. For example, single use cameras are returned to developers after use - this forms a natural collection point. Similarly, laser printers are serviced by local computer firms, and they collect empty toner cartridges.

1.5 Existing Literature and this paper

There is a large body of literature motivated by defense applications in the area of repairable inventories, and is covered well by the survey by V D R Guide. Most remanufacturing literature in Operations Management has focussed on inventory policies for the OEM, looking at aspects of reordering policies, order release mechanisms, yield uncertainty of the returned items. There are studies for cost structures in specific industry, and some abstraction of these cost structures , for example by Geraldo Ferrer. There is almost no economic or game theoretic literature in this area.

All existing work in this area consider a limited part of the logistics chain, for instance the production function, or the inventory function. Recent work by Toktay et al considers the cycle that the product travels and models this as a closed queuing network. In no paper is the issue of local remanufacturers considered.

There is substantial economic/strategy literature on the principal agent problem.

This paper studies the nature of incentives for OEMs to remanufacture the items when other remanufacturers also operate. Consumer demand is modeled explicitly. We use a two-period sequential entry model, and examine the optimal prices and quantities. We also model the effect of losses in the reverse logistics chain.

The main contribution of this paper is to put the remanufacturing industry in a competitive framework.

In this section we motivate the presentation. Section 2 will describe the model and the notation. In Section 3 we formulate the model and describe the solution space partition. The model is examined with a linear demand function in section 4, along with some results. Finally, Section 5 will conclude this paper.

2 Model Description

Two players are interested in a remanufacturable product. The first player is the OEM, who actually manufactures the item. The other player, called the local player, is an aggregation of the many small operators who remanufacture and sell the product locally.

The virgin product and the remanufactured product may be functionally equivalent. However, customers can distinguish between the product remanufactured by the OEM, and that remanufactured by the local remanufacturer. This means that the demand curve may be different for the locally remanufactured items and the OEM manufactured (or remanufactured) item. There is no loss in remanufacturing – all returned items (called shells) can be remanufactured and sold in the second period.

Since the local remanufacturer operates on a smaller scale, his unit cost of remanufacture may be more that that for the OEM, but of the same order of magnitude. The cost of manufacture is a few times the cost of remanufacture.

There are two periods. In the first period the OEM manufactures and sells items. It charges a price, and the quantity sold is determined by the demand function. (The Local player does not operate in the first period[3]).

[3] This is motivated by the fact that after introduction of a product, there is some time before the local remanufacturers "learn" how to gather shells and remanufacture them.

In the second period, a fraction of the shells is returned. We can interpret this fraction as loss in the reverse logistics chain[4]. A local remanufacturer can take back these shells and sell them after remanufacturing (at a lower price) in order to steal demand from the OEM. The OEM also takes back these items, and remanufactures them.

During the second period, the two players only obtain the amount that they can sell. The local remanufacturer cannot sell more than the amount returned since there are only that many shells. Moreover, the local remanufacturer, being closer to the point of consumption, gets the first chance at the shells. Any amount that remains may go to the OEM. If the OEM wants to sell more, it must manufacture them from new material. The price that the OEM charges in the second period cannot be different for the two types since they cannot be distinguished.

Both players attempt to maximize the undiscounted sum of profits for the two periods.

In this model, there are no observability issues – both players know everything.

2.1 Notation

The decisions facing the remanufacturer include

a) price of the item in the first period

b) price of the item in the second period.

The decisions facing the third party remanufacturer are

a) price of the item in the second period.

4 For a slightly different formulation, we can reinterpret this fraction as an obsolescence factor.

The following notation is used in the paper

p_{O1} : price charged by OEM in the first period

p_{O2} : price charged by OEM in the second period

p_{L2} : price charged by local firm in the second period

The parameters of the model are

c : cost of manufacturing the original product

r_O : cost of remanufacturing the original product, for the OEM

r_L : cost of remanufacturing the original product, for the local firm

α : fraction of the products returned in the second period, $\alpha \in [0,1]$

The demand for the product is described by the vector $D(p_O, p_L)$ where p_O is the price charged by the OEM, and p_L is the price charged by the local remanufacturer. The prices are necessarily restricted to the positive domain.

Some boundary conditions on the demand function in words are:

- For constant p_O, with increasing p_L, D_L decreases, while D_O increases but by less so that total D goes down.
- For constant p_L, with increasing p_O ($p_O > p_L$ according to the previous condition), D_O decreases, while D_L increases but by less so that total D goes down.
- For the first period (when only the OEM operates, we may assume that the local manufacturer "charges" a high enough price so that effectively the only demand is determined by the price charges by the OEM. This ensures a stationary demand functions for the two periods.

3 Formulation

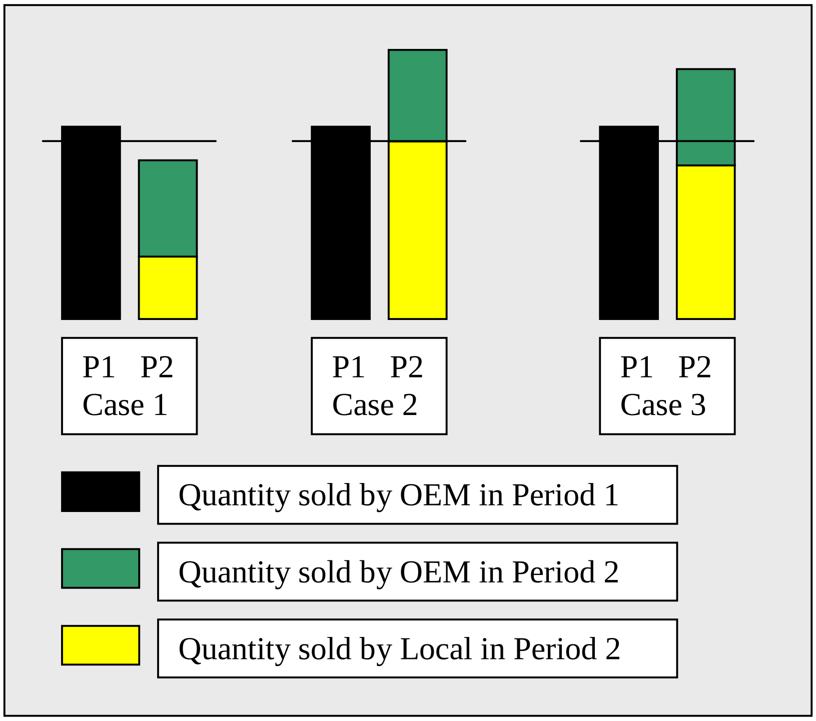
3.1 Solution space partition

There are a large number of parameters in this optimization. We can split all the solutions into three. (Quantity returned for remanufacture is α times quantity sold in the first period by player O) (Note that this partition depends on the cost of manufacture being more than the cost of remanufacture $c > r_O$ since otherwise the OEM will prefer to manufacture rather than manufacture in partitions 1 and 3)

Case 1: When the total quantity sold in the second period by O and L together is less than the quantity returned, i.e. $D_O(p*_{O2}, p*_{L2}) + D_L(p*_{O2}, p*_{L2}) \leq \alpha D_O(p*_{O1}, \infty)$. O only manufactures.

Case 2: When the quantity sold by L in the second period equals the quantity returned, i.e. $D_L(p*_{O2}, p*_{L2}) = \alpha D_O(p*_{O1}, \infty)$. O only remanufactures.

Case 3: When the quantity sold by L in the second period is less than the quantity returned, i.e. $D_L(p*_{O2}, p*_{L2}) < \alpha D_O(p*_{O1}, \infty)$ but the total quantity sold by both in the second period is more than the quantity returned, $D_O(p*_{O2}, p*_{L2}) + D_L(p*_{O2}, p*_{L2}) > \alpha D_O(p*_{O1}, \infty)$. O both manufactures and remanufactures.



3.2 The Model

$$\begin{aligned}\max_{p_{O1},p_{O2}}\ & \pi_O^1(p_{O1}) \\ & + I_{\text{Case I}}.\pi_O^{2\,I}(p_{O2}, p*_{L2}) \\ & + I_{\text{Case II}}.\pi_O^{2\,II}(p_{O2}, p*_{L2}) \\ & + I_{\text{Case III}}.\pi_O^{2\,III}(p_{O2}, p*_{L2}) \end{aligned} \qquad \cdots(1)$$

$$\text{s.t. } p*_{L2} \in \arg\max_{p_{L2}} \left[\pi_L^2(p_{L2})\right] \qquad \cdots(2)$$

$$\pi_L^2(p_{L2}) > 0 \qquad \cdots(3)$$

where the $\pi_{\text{player}}^{\text{period partition}}(.)$ refer to the profits, and can be explicitly written as

$$\pi_O^1(p_{O1}) = D_O(p_{O1}, \infty)(p_{O1} - c)$$

$$\pi_O^{2\,I}(p_{O2}, p*_{L2}) = \{D_O(p_{O2}, p*_{L2})(p_{O2} - r_O)\}$$

$$\pi_O^{2\,II}(p_{O2}, p^*_{L2}) = \{D_O(p_{O2}, p^*_{L2})(p_{O2} - c)\}$$

$$\pi_O^{2\,III}(p_{O2}, p^*_{L2}) = \begin{Bmatrix} [\alpha D_O(p_{O1}, \infty) - D_L(p_{O2}, p^*_{L2})](p_{O2} - r_O) \\ + [D_O(p_{O2}, p^*_{L2}) - \alpha D_O(p_{O1}, \infty)](p_{O2} - c) \end{Bmatrix}$$

$$\pi_L^2(p_{L2}) = \min[D_L(p_{O2}, p_{L2}), \alpha D_O(p_{O1}, \infty)].(p_{L2} - r_L)$$

Here I.. are indicator functions used to decide which partition it is in. In addition, note that in $\pi_L^2(p_{L2})$, the local remanufacturer cannot use up more than the quantity that was returned (hence the min()).

The formulation is similar to the principal-agent formulation[5], but with significant differences[6]. It is also similar to sequential entry games between two manufacturers (Stackelberg games)[7].

3.3 Partition 1

We can simplify the optimization if we restrict ourselves to each case in turn. Thus, first let us take partition 1. The optimization is now

$$\underset{p_{O1},p_{O2}}{\text{Max}}\; D_O(p_{O1}, \infty)(p_{O1} - c) + D_O(p_{O2}, p^*_{L2})(p_{O2} - r_O) \qquad \cdots(12)$$

s.t.

$$D_O(p_{O2}, p^*_{L2}) + D_L(p_{O2}, p^*_{L2}) \le \alpha D_O(p_{O1}, \infty) \qquad \cdots(13)$$

$$p^*_{L2} \in \arg\max_{p_{L2}}[D_L(p_{O2}, p_{L2}).(p_{L2} - r_L)] \qquad \cdots(14)$$

---

5 The principal tries to maximize his utility (i.e. the objective function) based on the behavior of the agent. The agent maximizes his own utility – the "rationality constraint" (i.e. constraint (2)). In addition, the agent gets more than his reservation (which is assumed to be 0)– the "incentive compatibility constraint" (constraint (3)).

6 It is different from a principal-agent formulation since both the principal and the agent decide on prices. Thus, in spite of complete observability, it is possible that the agent still makes a profit more than 0. This is not true of the standard principal-agent formulation where complete observability leads to the agent getting his reservation utility (the first best solution).

$$D_L(p_{O2}, p_{L2}).(p_{L2} - r_L) > 0 \qquad \cdots(15)$$

Note that in this case we have the added constraint (13), which represents the solution space partition.

3.4 Partition 2

Similarly, for the second partition we have

$$\underset{p_{O1}, p_{O2}}{\text{Max}}\ D_O(p_{O1}, \infty)(p_{O1} - c) + D_O(p_{O2}, p^*_{L2})(p_{O2} - c) \qquad \cdots(16)$$

s.t.

$$D_L(p_{O2}, p^*_{L2}) = \alpha D_O(p_{O1}, \infty) \qquad \cdots(17)$$

$$p^*_{L2} \in \arg\max_{p_{L2}} [\alpha D_O(p_{O1}, \infty).(p_{L2} - r_L)] \qquad \cdots(18)$$

$$\alpha D_O(p_{O1}, \infty).(p_{L2} - r_L) > 0 \qquad \cdots(19)$$

(17) represents the partition.

3.5 Partition 3

Finally, Partition 3 leads us to

$$\begin{aligned}\underset{p_{O1}, p_{O2}}{\text{Max}}\ & D_O(p_{O1}, \infty)(p_{O1} - c) \\ & + [\alpha D_O(p_{O1}, \infty) - D_L(p_{O2}, p^*_{L2})](p_{O2} - r_O) \\ & + [D_O(p_{O2}, p^*_{L2}) - \alpha D_O(p_{O1}, \infty)](p_{O2} - c) \qquad \cdots(20)\end{aligned}$$

s.t.

$$D_O(p_{O2}, p^*_{L2}) + D_L(p_{O2}, p_{L2}) > \alpha D_O(p_{O1}, \infty) \qquad \cdots(21)$$

$$D_L(p_{O2}, p^*_{L2}) < \alpha D_O(p_{O1}, \infty) \qquad \cdots(22)$$

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[7] Here again, the differences lie in the multi-period nature of the game and the fact that although the OEM starts first, in the second period the Local remanufacturer decides first how much to remanufacture.

$$p^*_{L2} \in \arg\max_{p_{L2}} \left[D_L(p_{O2}, p_{L2}).(p_{L2} - r_L)\right] \quad \cdots(23)$$

$$D_L(p_{O2}, p_{L2}).(p_{L2} - r_L) > 0 \quad \cdots(24)$$

Two extra constraints (21 and 22) describe this partition.

3.6 Propositions

For this section, I assume that the demand functions are continuous and strictly monotonic in p_O and p_L.

Proposition 3.1: p^*_{L2} is continuous in p_{O1} and p_{O2}.

Proposition 3.2: The objective function is continuous in p_{O1} and p_{O2}.

Proposition 3.3: For a sufficiently nice demand function, $\pi_L^2(p_{L2})$ has a unique maximum.

4 Linear demand vector

Linear demand functions are common in literature. The linear demand vector is an extension.

A linear demand vector common in economic literature (e.g. Stackelberg games where two firms sequentially enter the market) is the following

$$D_O(p_O, p_L) = A_O - B_O p_O + C_O p_L \quad \cdots(101)$$

$$D_L(p_O, p_L) = A_L - B_L p_L + C_L p_O \quad \cdots(102)$$

This captures the effect of the other firm's price upon the quantity demanded by each firm. Note that since total demand must decrease for any increase in price, (B_O-C_L) must be positive, as well as (B_L-C_O)[8].

We can solve simultaneously for both demand expressions being 0. This will give us the "natural" limits on p_O and p_L as

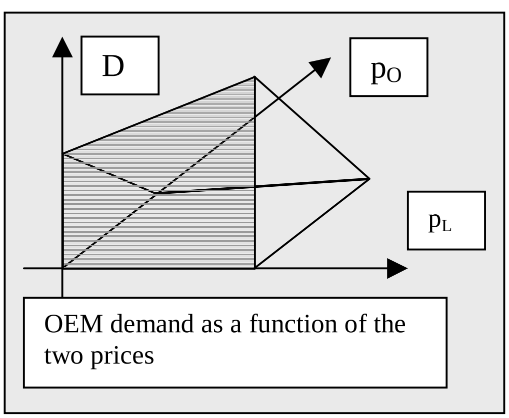
$$p_O \leq \frac{B_L A_O + C_O A_L}{B_O B_L - C_O C_L} \qquad \cdots(103)$$

$$p_L \leq \frac{B_O A_L + C_L A_O}{B_O B_L - C_O C_L} \qquad \cdots(104)$$

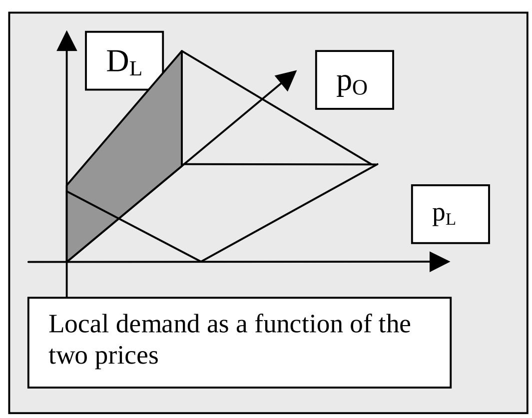
In addition, we need the demand function that the OEM faces in the first period. In the first period, only the OEM sells. Hence, we can take this as $D_O(p_O, p_{L\,max})$

$$\begin{aligned} D_O(p_O,\infty) &= D_O(p_O, p_{L\max}) \\ &= \left(A_O + C_O \frac{B_O A_L + C_L A_O}{B_O B_L - C_O C_L}\right) - B_O p_O \\ &\equiv A'_O - B_O p_O \qquad \cdots(105) \end{aligned}$$

Thus, we can visualize the demand functions as



OEM demand as a function of the two prices



Local demand as a function of the two prices

8 At first glance it does not make sense that D_L can increase indefinitely if p_O increases. However, if we choose the C parameters to be smaller than the B parameters (i.e. the drop per unit of own price increase is more than the rise per unit of other price decrease), we see that optimal price selected by O does not exceed some maximum. This makes sense, since the total quantity cannot increase if any one player increases his price. This puts natural bounds on p_O, and hence we avoid the "problem".

4.1 Propositions

We have $\{B_O, B_L\} > \{C_O, C_L\}$ and $\{A_O, A_L\} > \{B_O, B_L, C_O, C_L\}$.

Proposition 4.1: The optimization reduces to a quadratic programming problem in each partition. Each objective function has a negative definite hessian. Hence one local optimum exists in each partition.

Proposition 4.2: If any partition does not have an interior local optimum then the optimal solution does not lie in the interior of that partition.

Proposition 4.3: Partition 3 does not have an interior solution.

Proposition 4.4: If the Partition 1 constraint holds strictly, Partition 1 is an interior solution obtained by the first order conditions. Otherwise there is a boundary solution. Similarly if the Partition 2 constraint holds strictly, Partition 2 has an interior solution. Otherwise it has a boundary solution.

4.5 Solution Methodology

While the solution is explicit for each partition, it is difficult to figure out whether the corresponding constraint is satisfied or violated. Hence, from the form of the solution for a specific partition, we can conduct static analysis conditional on the solution still satisfying the specific constraint.

The optimal solution in each partition is a subgame perfect Nash equilibrium of the game. In certain parameter space regions both equilibria may exist. Hence the solution methodology consists of checking the respective constraint in partitions 1 and 2. If the constraint is strictly satisfied, there is an equilibrium in the interior of that partition. Otherwise, it is at the boundary.

4.6 Results

In this section results from the analytic solution and numerical results are used to derive insights.

…

5 Conclusion

Appendix

A.1 Proofs of Propositions

Proposition 3.1

The demand function is continuous, and hence $\min\left[D_L(p_{O2}, p_{L2}), \alpha D_O(p_{O1}, \infty)\right]$ is continuous in p_{L2}. Hence, $\pi_L^2(p_{L2})$ is continuous in p_{L2}.

Proposition 3.2

At the boundary of partition 1 and 3, the value of p_{L2} is does not change, since the L quantity is still less than the total returns. Hence the OEM profit is continuous at this boundary.

At the boundary of partitions 2 and 3, the value of p_{L2} is continuous even though it is constrained in one partition by the amount of returns. Hence the OEM profit is continuous at this boundary.

Proposition 3.3

With a strictly monotonic demand function, if p_{L2} is below the threshold, it is constrained by the amount of returns, and hence L profit is strictly increasing in p_{L2} till this threshold. Above it, if $D_L(p_{O2}, p_{L2}).(p_{L2} - r_L)$ decreases then the unique p_{L2} is the threshold value. If $D_L(p_{O2}, p_{L2}).(p_{L2} - r_L)$ continues to increase, then if

$\frac{\partial D_L(p_{O2}, p_{L2})}{\partial p_{L2}}.(p_{L2} - r_L) + D_L(p_{O2}, p_{L2}) = 0$ has a unique solution, p_{L2} is unique.

Proposition 4.1

4.1 Partition 1

Let us now apply the linear demand function to partition 1.

Note that in all cases (15) is satisfied trivially as long as $p_{L2}>r_L$ since demand is non-negative.

For (14), we now have

$$p^*_{L2} = \underset{p_{L2}}{\arg\max}\left[(A_L - B_L p_{L2} + C_L p_{O2})(p_{L2} - r_L)\right]$$
$$\text{s.t.} \quad p_L > r_L$$
$$= \begin{cases} \dfrac{A_L + C_L p_{O2} + B_L r_L}{2B_L} & \text{if} \quad p_{O2} > \dfrac{B_L r_L - A_L}{C_L} \\ r_L & \text{otherwise} \end{cases} \qquad \cdots(111)$$

If p^*_{L2} is r_L, then L is not selling anything – his profit is nil, and he is indifferent to staying in the game. This means that the only player is OEM. Hence we may now take the additional constraint implied on p_{O2} from (111). But $\dfrac{B_L r_L - A_L}{C_L}$ is negative for all problems where Local has some non zero profit prices. Hence it is not necessary to include this constraint.

Substituting for p^*_{L2}, and rearranging,

$$\begin{aligned} \underset{p_{O1},p_{O2}}{\text{Max}} \quad & (-B_O)p_{O1}^2 + \left(-B_O + \frac{C_O C_L}{2B_L}\right)p_{O2}^2 \\ & + (A'_O + B_O c)p_{O1} + \left(A_O + \frac{C_O A_L}{2B_L} + \frac{C_O r_L}{2} + B_O r_O - \frac{C_O C_L r_O}{2B_L}\right)p_{O2} \\ & + \left(A'_O c - A_O r_O - \frac{C_O A_L r_O}{2B_L} - \frac{C_O r_L r_O}{2}\right) \qquad \cdots(112) \end{aligned}$$

s.t.

$$(A_L + A_O - \alpha A'_O) + (C_L - B_O + \alpha B_O)p_{O1} + (C_O - B_L)p_{O2} < 0 \qquad \cdots(113)$$

This is a quadratic programming problem.

4.1 Partition 2

(19) is satisfied trivially as long as $p_{L2}>r_L$ since demand is non-negative. We obtain p^*_{L2} very easily by using (17).

$$A_L - B_L p^*_{L2} + C_L p_{O2} = \alpha(A'_O - B_O p_{O1})$$
$$\text{or} \quad p^*_{L2} = \frac{A_L - \alpha A'_O + \alpha B_O p_{O1} + C_L p_{O2}}{B_L} \qquad \cdots(121)$$

Incorporating the linear demand function into the objective function and simplifying, we obtain

$$\begin{aligned} \underset{p_{O1},p_{O2}}{\text{Max}} \quad & -B_O p_{O1}^2 + \left(-B_O + \frac{C_O C_L}{B_L}\right)p_{O2}^2 + \frac{\alpha B_O C_O}{B_L} p_{O1} p_{O2} \\ & + \left(A'_O + B_O c - \frac{\alpha B_O C_O c}{r_L}\right)p_{O1} + \left(A_O + \frac{A_L C_O}{B_L} - \frac{\alpha A'_O C_O}{B_L} + B_O c - \frac{C_L C_O c}{B_L}\right)p_{O2} \\ & + \left(-A'_O c - A_O c - \frac{A_L C_O c}{B_L} + \frac{\alpha A'_O C_O c}{B_L}\right) \qquad \cdots(122) \end{aligned}$$

s.t.

$$(\alpha A'_O + B_L r_L - A_L) - \alpha B_O p_{O1} - C_L p_{O2} < 0 \qquad \cdots(123)$$

This, again, is a quadratic program.

4.1 Partition 3

For the third partition, we again have the same form for p^*_{L2} as given in Partition 1 by equation (111). We substitute this into the formulation. Arranging the coefficients, we obtain

$$\begin{aligned} \underset{p_{O1},p_{O2}}{\text{Max}} \quad & -B_O p_{O1}^2 \\ & + (A'_O + B_O c + \alpha B_O c - \alpha B_O r_O)p_{O1} + \left(-B_O c + \frac{C_L C_O c}{2B_L} + B_O r_O - \frac{C_L C_O r_O}{2B_L}\right)p_{O2} \\ & + \left(A_O c - A'_O c + \alpha A'_O c + \frac{A_L C_O c}{2B_L} + \frac{C_O r_L c}{2} - A_O r_O + \alpha A'_O r_O - \frac{A_L C_O r_O}{2B_L} - \frac{C_O r_L r_O}{2}\right) \\ & \qquad \cdots(131) \end{aligned}$$

s.t.

$$(\alpha A'_O - A_L - A_O) + (B_O - \alpha B_O - C_L)p_{O1} + (B_L - C_O)p_{O2} < 0 \qquad \cdots(132)$$

$$(A_L - \alpha A'_O) + (\alpha B_O - C_L)p_{O1} - B_L p_{O2} < 0 \qquad \cdots(133)$$

This is the third quadratic program.

Proposition 4.2

Consider the partitions in terms of p_{O1} and p_{O2}. Partition 1 borders Partition 3 and Partition 3 borders Partition 2 as well. (The other borders for Partition 1 and 2 are cases where either demand or price is 0.) Since the OEM profit is continuous in the two variables, if any partition has a boundary solution, it implies that the neighbouring partition may be more desirable. (The optimal boundary solution is never the 0 demand or 0 price boundary for demand functions under consideration.)

Proposition 4.3

The objective function in partition 3 has no quadratic or cross term in p_{O2}. In addition the coefficient for p_{O2} is not 0. Hence the optimal solution must be at a boundary.

Proposition 4.4

For partition 1, the first order conditions give us the following results

$$p^*_{O1} = \frac{1}{2}\left(\frac{A'_O}{B_O} + c\right) \text{ and } p^*_{O2} = \frac{1}{2}\left(\frac{2B_L\left(A_O + C_O r_L\right) + C_O\left(A_L - B_L r_L\right)}{2B_O B_L - C_O C_L} + r_O\right),$$

with OEM profit given by

$$\pi_O = \frac{1}{8}\left(\frac{2\left(A'_O - B_O c\right)^2}{B_O} + \frac{\left(2A_O B_L + A_L C_O + B_L C_O r_L - 2B_L B_O r_O + C_L C_O r_O\right)^2}{B_L\left(2B_O B_L - C_O C_L\right)}\right)$$

as long as the following constraint holds

$$\frac{1}{2}\left(\begin{array}{l} 2A_L + 2A_O + \left(A'_O + B_O c\right)\left(\dfrac{C_L}{B_L} - 1\right) + \alpha\left(A'_O - B_O c\right) \\ + \left(C_O - B_L\right)\left(\dfrac{2B_L\left(A_O + C_O r_L\right) + C_O\left(A_L - B_L r_L\right)}{2B_O B_L - C_O C_L} + r_O\right) \end{array}\right) < 0$$

For partition 2, similarly, we have

$$p^*_{O1} = c + \frac{\begin{pmatrix} 2B_L\left(B_OB_L - C_OC_L\right)\left(A'_O - B_Oc\right) \\ + \alpha B_OB_LC_O\left(A_O - B_Oc\right) \\ - \alpha B_OC_O{}^2\left(\alpha\left(A'_O - B_Oc\right) - \left(A_L + cC_L\right)\right) \end{pmatrix}}{4B_OB_L\left(B_OB_L - C_OC_L\right) - \alpha^2B_O{}^2C_O{}^2}$$

$$\text{and } p^*_{O2} = c + \frac{B_L\begin{pmatrix} 2B_L\left(A_O - B_Oc\right) \\ + 2C_O\left(A_L + C_Lc\right) \\ - \alpha B_OC_O{}^2 \end{pmatrix}}{4B_OB_L\left(B_OB_L - C_OC_L\right) - \alpha^2B_O{}^2C_O{}^2}$$

with OEM profit given by

$$\pi_O = \frac{\begin{pmatrix} B_OB_L{}^2\left(A_O{}^2 + A'_O{}^2 - 2B_Oc\left(A_O + A'_O\right) + 2B_O{}^2c^2\right) \\ + B_LC_O\begin{pmatrix} B_O\left(A_O - B_Oc\right)\left(2A_L - \alpha A'_O + \alpha B_Oc\right) \\ - C_L\left(A'_O{}^2 - 2\left(A_O + A'_O\right)B_Oc + 3B_O{}^2c^2\right) \end{pmatrix} \\ + B_OC_O{}^2\left(A_L + C_Lc\right)\left(A_L - \alpha A'_O + \alpha B_Oc + C_Lc\right) \end{pmatrix}}{4B_OB_L\left(B_OB_L - C_OC_L\right) - \alpha^2B_O{}^2C_O{}^2}$$

as long as

$$\frac{B_L\begin{pmatrix} \left(\alpha\left(A'_O - B_Oc\right) - 2A_L\right)\left(2B_OB_L - C_OC_L\right) \\ - 2C_L\left(A_OB_L + \left(B_OB_L - C_OC_L\right)c\right) \\ + 4B_L\left(B_OB_L - C_OC_L\right) \\ - \alpha^2B_OC_O\left(A_O - B_Oc + C_Or_L\right) \end{pmatrix}}{4B_L\left(B_OB_L - C_OC_L\right) - \alpha^2B_OC_O{}^2} < 0$$

The boundary solution for these two partitions can be similarly obtained.

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